Size and Capitalization Adjustments for Market-Based Pricing Multiples

By Derek Zweig, CFA, FRM, and Adam Luke, PhD

In this article, we describe an approach for adjusting market-based equity- and firm-level pricing multiples for differences in size (market capitalization) and capital structure. All size and capitalization adjustments are channeled through the modified capital asset pricing model, which is relied upon for estimating cost of equity. The impact of size and capitalization on the cost of equity manifests itself through the size premium and the systematic risk coefficient, respectively. The practitioners' method is used to incorporate leverage into the systematic risk coefficient.

Note: References are listed on page 17. A glossary of key variables appears on page 10.

I. Introduction

The ideal methodology to value an asset is to observe the prices paid for that asset in the open market. However, when valuing shares of a privately held company, no such marketplace exists. Valuation theory thus includes consideration of three commonly accepted approaches to value: the market approach, the income approach, and the asset approach. Each approach contains various valuation methodologies. The choice of which approaches and methodologies to utilize depends on the specific facts and circumstances of the valuation.

Under the market approach, the value of a company is estimated based on pricing relationships associated with observable market dealings involving similar companies. These observations make it possible to estimate the value of shares that have no active market. The market approach includes two primary methods: the guideline public company method (GPCM) and the guideline transaction method. This article focuses on the GPCM.

Application of the GPCM requires the availability of data from public companies that are reasonably comparable to the private company being valued. The criteria for comparability in the selection of publicly traded guideline companies include business operational characteristics, growth patterns, relative size, earnings trends, markets served, and risk characteristics. Once a guideline group is selected, various pricing multiples may be obtained and used to estimate a value for the subject private company.

Pricing multiples have useful interpretations that lead to more general treatment. We define two types of cash flows: (1) cash flows available to the whole firm (debt and equity holders) (c^F) and (2) cash flows available only to equity holders (c^e).[1] For simplicity, this introduction emphasizes pricing multiples using equity cash flows, though the methodology may also be applied to pricing multiples on enterprise-level cash flows (discussed in Section III).

Using Company i's market value of equity (V_i^e) and equity cash flows (c_i^e), one may recover the observable equity pricing multiple, M_i^e:

$$M_i^e = \frac{V_i^e}{c_i^e} \tag{1}$$

Perhaps Company i is a reasonable guideline company for Company j. Applying the pricing multiple computed for Company i to the equity cash flows of Company j (c_j^e), one obtains an estimate for the value of Company j's equity (V_j^e):

$$c_j^e * M_i^e = V_j^e \tag{2}$$

Equation 2 details how a pricing multiple may provide an indication of value for Company j using observable public information from Company i. This simplistic form ignores a time index for each of the inputs and outputs, but demonstrates the concept of a pricing multiple.

As pointed out by Gordon and Shapiro (1956), the value of any asset may be estimated by taking an infinite sum of the cash flows generated by the asset, each discounted to present at a rate commensurate with the rate of profit.[2] Using equity cash flows, we have

$$V^e = \sum_{t=1}^{\infty} \frac{c_t^e}{(1+k^e)^t} \tag{3}$$

1 Cash flows available to the whole firm commonly include revenue, gross profit, earnings before interest and taxes (EBIT), and earnings before interest, taxes, depreciation, and amortization (EBITDA). Cash flows available to equity holders commonly include net income and gross cash flow (net income before depreciation and amortization).

2 We do not mean here the asset's actual rate of profit. Rather, the rate of profit represents the relation between the present value of an asset and expected future cash flows.

The discount rate (k^e) in Equation 3 is known as the cost of equity. It represents the rate of return required for an investor to justify investing in the equity of a company, given its current value and risk profile. An alternative, though equivalent, interpretation frames cost of equity as the opportunity cost of capital, i.e., the return one should expect to receive for investing in the equity of a different company with a comparable risk profile.

Under certain assumptions[3] about cash flow growth (g_{c^e}), the identity in Equation 3 can be expressed as the well-known Gordon growth model:

$$V^e = \frac{c^e}{k^e - g_{c^e}} \tag{4}$$

Writing Equation 4 as

$$V^e = c^e * \frac{1}{k^e - g_{c^e}} \tag{5}$$

and dividing both sides by the equity cash flow, we can see that

$$\frac{V^e}{c^e} = \frac{1}{k^e - g_{c^e}} \tag{6}$$

From Equation 1, it is clear that the left-hand side of Equation 6 is equivalent to the equity-level pricing multiple:

$$M^e = \frac{1}{k^e - g_{c^e}} \tag{7}$$

Defining the equity-level capitalization rate (z^e) as

$$z^e = k^e - g_{c^e} \tag{8}$$

we can now write the pricing multiple as

$$M^e = \frac{1}{z^e} \tag{9}$$

This interpretation of pricing multiples is useful for facilitating more accurate comparison of one company's price with another. While only the pricing multiple is observable for publicly traded companies, one may reduce the pricing multiple down to Equation 9 to back out the implied equity-level capitalization rate. In applying the pricing multiple from Company i to the performance of Company j, one may change components of the capitalization rate (per Equation 8) to arrive at a pricing multiple tailored to Company j. In particular, differences in risk between Company i and Company j can be explicitly incorporated through adjustments to k^e.

The remainder of this article will discuss different frameworks for estimating, and adjusting, the cost of equity to account for differences in size and capital structure. In Section II, we review relevant literature regarding the calculation of cost of equity and the importance of size and capitalization in this calculation. We detail our approach to adjusting pricing multiples for size in Section III and for capitalization in Section IV. Readers may refer to the glossary on page 10 for descriptions of key variables used. Section V provides examples that illustrate the pricing multiple adjustments applied in this article.

II. Concept Overview

Pioneering work from several authors[4] led to early adoption of the capital asset pricing model (CAPM) as the leading approach to explain cost of equity. Under several restrictive assumptions,[5] the CAPM says[6]

$$E(r_i) = r_f + \beta_i(r_m - r_f) \tag{10}$$

Here, the expected return for Company i (r_i) is a function of the risk-free rate (r_f), the excess return of the market ($r_m - r_f$), and Company i's systematic risk coefficient, β_i. $\beta_i = 0$ indicates that Company i has zero systematic risk, while $\beta_i = 1$ indicates that Company i's systematic risk is identical to that of the market as a whole. The market return (r_m) is often estimated using a broad equity index, such as the S&P 500.

While this approach to estimating the cost of equity is straightforward, decades of empirical work suggest that the CAPM consistently underestimates equity returns.[7]

Banz (1981) was the first to note that the degree of underestimation seemed to coincide with different measures of company size. Banz used the simple linear functional form,

$$E(r_i) = \gamma_0 + \gamma_1\beta_1 + \gamma_2\left[\frac{\phi_i - \phi_m}{\phi_m}\right] \tag{11}$$

3 To recover the identity in Equation 4, we assume that cash flows grow at a constant rate, and that growth occurs in continuous time. We additionally assume that the cost of equity is constant.

4 Treynor (1961), Sharpe (1964), Lintner (1965a), Lintner (1965b), Mossin (1966), and Black (1972), to name a few.

5 Investors: (a) have identical and complete information, (b) have homogeneous expectations, (c) are rational and risk-averse, (d) maximize utility, (e) diversify investments, and (f) are price takers. Additionally, all assets are assumed perfectly divisible and liquid, trading frictions (taxes and transaction costs) are assumed not to exist, and borrowing and lending is assumed to be available in unlimited amounts at the risk-free rate.

6 As noted in Section I, the cost of equity may be interpreted as the return one should expect to receive for investing in the equity of a different company with a comparable risk profile. Both $E(r)$ and k^e are used interchangeably to refer to the cost of equity.

7 Basu (1977), Banz (1981), Ostermark (1990), Fama and French (2004), Choudhary and Choudhary (2010), Agrawal, Mohapatra, and Pollak (2011), and Yasmeen et al. (2012), to name a few. Note that these studies cover both developed and emerging markets.

where: r_i is the equity return for Company i, γ_0 is the risk-free rate (i.e., the return on a zero-beta portfolio), γ_1 is the expected market risk premium ($r_m - \gamma_0$), ϕ_i is the market value of equity of Company i, ϕ_m is the average market value of equity for the market as a whole, and γ_2 is the constant measuring the contribution of ϕ_i to the expected equity return. If there is no size premium, then $\gamma_2 = 0$, and the equation would reduce to traditional CAPM.

Banz found that, on average, smaller firms have higher risk-adjusted returns than larger firms. This size premium was also found to be nonlinear, accelerating in magnitude as companies get increasingly small.

A number of critiques followed, with a host of alternative explanations arising for the observed size premium. As Banz himself notes, "It is not known whether size *per se* is responsible for the [size premium] or whether size is just a proxy for one or more true unknown factors correlated with size."[8]

Some critics have noted that small, particularly troubled, firms[9] tend to trade less frequently, resulting in a lagged price relation to the market relative to larger companies. Additionally, as the market value of a troubled firm's equity is bid down, it may begin to trade like a call option. Both of these properties could result in an overestimation of size premium for smaller firms due to an underestimated beta.[10] Practitioners have overcome this critique in several ways. Asness et al. (2017) found that the size premium is salvaged when controlling for troubled firms. Other practitioners suggest alternative measures of beta,[11] such as the "sum beta." The sum beta methodology adjusts Equation 10 to better capture lagged effects from the prior time period (i.e., time period $t-1$):

$$E(r_t) = r_{f,t} + \beta_t(r_{m,t} - r_{f,t}) + \beta_{t-1}(r_{m,t-1} - r_{f,t-1})$$
$$\text{Sum Beta} = \beta_t + \beta_{t-1} \tag{12}$$

Calculating beta using annual (rather than monthly) return data can have a similar effect as using sum beta, in that it captures lagged manifestation of systematic risk through a higher beta.

Data artifacts, such as bid-ask bounce and delisting bias, may also be responsible for mismeasurement of the size premium.[12]

The existence of bid-ask spreads for publicly traded equities may add a bias to measured returns, particularly for smaller, less liquid equities with larger spreads. This bias, known as bid-ask bounce, results from the movement from bid to ask (and back to bid) in a sequence of trades, causing a measured rate of return that is larger than the movement from the same ask to the same bid. With trades being executed randomly at either bid or ask price, a small bias can creep into the return measurement. This problem is most extreme if using daily returns. Calculating returns over a larger horizon, such as monthly, mitigates the impact of bid-ask bounce.

The worse a company's financial performance, the more likely it is to be delisted from exchanges relative to companies with strong financial performance. Dropping these poorly performing companies from the data may cause the size premium to be inflated. This bias can be partially avoided by not limiting the data to exchange-traded companies, though survivorship bias may remain. Empirical evidence suggests that the delisting bias is relatively tiny (only about 20 basis points).

8 Banz (1981, 3) (abstract).
9 Troubled firms are those considered to be of low quality. Profitability, stability, and growth can all be considered proxies for quality.
10 Pratt and Grabowski (2014).
11 Ibid.
12 Ibid.

Glossary

The following glossary of variable definitions may assist readers in understanding the mathematical modeling in this article. The list is not exhaustive but covers all key variables.

V^e	Market value of equity
V^F	Market value of the firm (debt and equity)
c^e	Cash flow to equity holders
c^F	Cash flow to the firm (debt and equity holders)
M^e	Equity-level pricing multiple $\left(\frac{V^e}{c^e}\right)$
M^F	Firm-level pricing multiple $\left(\frac{V^F}{c^F}\right)$
k^e	Cost of equity
k^d	Cost of debt
g_c^e	Growth in cash flow to equity
g_c^F	Growth in cash flow to the firm
z^e	Equity-level capitalization rate $(k^e - g_c^e)$
z^F	Firm-level capitalization rate $(WACC - g_c^F)$
t	Time in years
x	Tax rate
r	Total equity return, inclusive of capital gains and dividends
r_f	The risk-free rate
$E(\cdot)$	The expectation operator, representing the average of the variable or function inside the parentheses
S	A size variable, including but not limited to revenue, enterprise value, or market capitalization
D	Debt capitalization
E	Equity capitalization

Subscripts i and j are used to indicate which firm is being referenced. Subscript i indicates Company i, while subscript j indicates Company j. The subscript m is used to indicate a market variable. The subscript t is used to index time. β and γ are used as model coefficients. Superscripts L and U indicate levered and unlevered variables, respectively.

Other critics have pointed out that sampling nuances may distort the size premium measurement. In particular, the time period examined can impact results, as can the "January effect."

After Banz (1981), the size premium seemed to disappear. It was believed this may have been due to market adaptation after the size premium had been revealed. Hou and van Dijk (2018) note that realized returns for small companies fell below expectations after the early 1980s due to negative cash flow shocks. These shocks were attributable to a large increase in public offerings in the 1980s. These newly public firms had profitability and survival rates below those of firms that went public in prior decades. After adjusting for these cash flow shocks, the size premium remained intact.

The January effect is another issue thought to distort measurements of the size premium.[13] The January effect is related to tax-loss selling that occurs at the end of each calendar year. Losses are recognized from the sale and then equity shares are repurchased in January. This acts to temporarily depress stock prices and can cause companies to be miscategorized in the size rankings. While important, such a critique only applies when market capitalization is the metric relied upon for measuring size. If other metrics are relied upon (e.g., annual revenue), the January effect will not bias the size premium calculation.

The last major critique regards liquidity. As was alluded to in our discussion of bid-ask bounce, smaller firms tend to be categorically less liquid than larger firms. This implies that investors may require a higher expected return to compensate them for taking on more liquidity risk.[14] Accordingly, we cannot reject the possibility that a liquidity premium is partially responsible for the size premium. To the extent smaller stocks are less liquid and investors incur larger transaction costs, investors may simply price a stock in a way to reimburse themselves for increased costs. There are, however, several reasons why adjusting discount rates (and corresponding pricing multiples) for liquidity may be inappropriate.[15]

As a component of marketability, liquidity is generally handled separately when valuing a closely held business. Nonetheless, expressing discount rates net of transaction costs would make them ill-suited for discounting projected before-transaction-cost cash flows. Furthermore, any

13 Rozeff and Kinney (1976); Haug and Hirschey (2006).
14 Torchio and Surana (2014); Asness et al. (2018).
15 Pratt and Grabowski (2014).

analyst making an adjustment for transaction costs must acknowledge that investors can extend the time horizon of their holdings, mitigating these costs.

While each of these critiques is legitimate, it is clear that adjustments to the measurement methodology can overcome almost all of them. Recent empirical work[16] reveals that as methodology has improved to avoid these measurement pitfalls, the size premium remains. This leads us to ask: If the premium is in fact valid, what practical and theoretical explanations account for its existence?

From a theoretical standpoint, there are several properties of small stocks that make their risks fundamentally different from large stocks.[17] Because investment data is more sparse for small stocks, and analyst coverage less robust, investors must exert more effort per dollar invested to perform adequate analysis of an investment opportunity. In conjunction with lower liquidity, investors may have more trouble diversifying away the risk of small stocks.

From a practical standpoint, small stocks do not enjoy many of the protections afforded to large firms.[18] It is much easier for large firms to protect their market share from new industry entrants, in turn protecting equity value in ways small firms cannot. Similarly, a larger resource base and superior access to capital markets allows larger firms to more easily enter new markets themselves and makes them more resistant to adverse economic fluctuations. Also, with more resources, large firms are able to spend more on research, advertising, and hiring, all of which provide them with greater influence on demand and the competitive landscape. Larger firms also tend to have deeper management teams and more fragmented customer bases than small firms, reducing concentration risks.

Each of these theoretical and practical differences between small and large firms works to increase the required return on an investment in small stocks. Of course, the size premium is not predictive, and cannot be relied upon for any particular holding period. In other words, the size premium tends to be cyclical. There are many historical periods in which small stocks significantly underperform large stocks. If this were not true, there would not actually be an increased risk associated with small firms. Hence, the cyclicality is a manifestation of the size risk, so it is not surprising that the size premium fluctuates across time, only stabilizing over longer time horizons.[19]

16 Asness et al. (2018), Ciliberti et al. (2017), Grabowski (2018), and Hou and van Dijk (2019), among others.
17 Pratt and Grabowski (2014).
18 Ibid.
19 Grabowski (2018).

The empirical importance of the size premium leads us to the modified CAPM as an improvement on the CAPM in measuring cost of equity:

$$k_i^e = E(r_i) = r_f + \beta_i(r_m - r_f) + \gamma S_i \tag{13}$$

Here, S_i is a proxy for the size[20] of Company i, and γS_i gives the size premium for Company i.

We conclude this section with a note on the systematic risk coefficient (β). This coefficient is not agnostic to a firm's capitalization. All things being equal, the more debt a firm has in its capital structure, the riskier its equity. In turn, riskier equity warrants a higher cost of equity. This increased risk from debt is channeled through the firm's beta. The higher a firm's debt capitalization, the more sensitive its equity to the excess return of the market. This can be demonstrated by introducing levered and unlevered beta.

A company's systematic risk is thought to consist of two components. The first component, operating risk, reflects uncertainty inherent in a firm's cost structure (i.e., fixed costs versus variable costs). The second component, financial risk, captures uncertainty attributable to a firm's capital structure (i.e., debt versus equity). A levered beta (β^L) incorporates both operating and financial risk.

Unlevering beta refers to the process of stripping out the effects of financial leverage (debt), leaving only the portion of systematic risk attributable to operating risk. Unlevered beta (β^U), also known as asset beta, can be thought of as a firm's systematic risk were it only equity financed.

A plethora of methods exist for unlevering beta, each containing a series of implicit assumptions, many of which regard the interaction of interest and taxes.[21] The most practical approach to derive unlevered beta is the practitioners' method.[22] Under this approach, unlevered beta may be found as follows:

$$\beta^U = \frac{\beta^L}{1 + \frac{D}{E}} \tag{14}$$

D and E refer to the debt capitalization and equity capitalization, respectively. Specifically, if a and b refer to the value of a company's equity and debt, respectively, then $E = \frac{a}{(a+b)}$ and $D = \frac{b}{(a+b)}$. This straightforward approach assumes that E and D are constant, meaning a firm's debt grows in proportion to its firm-level cash flows. This approach also assumes that the cost of equity, calculated using a firm's asset beta, is the appropriate discount rate to use when calculating the tax shield (stemming from interest on debt). This means the tax shield is discounted at the same rate as operating cash flows.

III. Size Adjustment Methodology

As demonstrated in Equation 9, the equity-level capitalization rate is a direct function of the cost of equity, which is in turn a function of the modified CAPM.[23] Substituting Equation 13 into Equation 9, we get the capitalization rate for Company i as a function of the modified CAPM:

$$\begin{aligned} z_i^e &= k_i^e - g_{c_i^e} \\ &= r_f + \beta_i(r_m - r_f) + \gamma S_i - g_{c_i^e} \end{aligned} \tag{15}$$

As noted in Section I, when applying pricing multiples derived from Company i to find the value of Company j, differences in risk between Company i and Company j can be explicitly incorporated through adjustments to k^e. Using the modified CAPM framework to estimate cost of equity, there are two variables through which adjustments may run: β and γS. In other words, one may alter the systematic risk and size premium of Company i to better match that of Company j.

Consider two realizations of size: S_i for Company i and S_j for Company j. Company i is a public company whose pricing multiples (and hence capitalization rates) are observable, while Company j is a closely held business. Company i is considered an appropriate public comparable for valuing Company j. To apply the size adjustment to Company i's equity pricing multiple, one simply replaces S_i with S_j in the capitalization rate equation. This recovers the equity-level capitalization rate for Company j:

$$z_j^e = r_f + \beta_i(r_m - r_f) + \gamma S_j - g_{c_i^e} \tag{16}$$

In turn, the equity-level capitalization rate can be inverted to provide the adjusted equity pricing multiple for Company j:

$$M_j^e = \frac{1}{z_j^e} \tag{17}$$

20 Common choices include market capitalization, revenue, EBITDA, total assets, and book value of equity.

21 The Hamada method, the Miles-Ezzell method, the Harris-Pringle method, and the Fernandez method, among others. While the implicit assumptions of these methods may be more or less desirable depending on the application, each requires more granular (and unobservable) company data than the practitioners' method.

22 Fernandez (2003).

23 As is the pricing multiple, since the pricing multiple is just an inversion of the capitalization rate (*see* Equation 9).

So far, we have emphasized equity cash flows. Accordingly, Equation 17 represents the size-adjusted pricing multiple valid for all pricing multiples wherein the cash flow stream is accessible only to equity owners. The use of k^e, the cost of equity, is only appropriate when the cash flow stream is accessible only to equity owners. If we instead emphasize some cash flow to the firm (c^F), then k^e is no longer appropriate. Rather, we must then use the weighted average cost of capital (WACC), defined as

$$WACC = k^e * E + k^d * (1 - x)D \tag{18}$$

Here, E and D continue to refer to the equity capitalization and debt capitalization, respectively, k^d represents the cost of debt, and x the tax rate. Rewriting Equation 5 using the WACC in place of the cost of equity and c^F in place of c^e, we recover the value of the firm (V^F):

$$V^F = c^F * \frac{1}{WACC - g_{c^F}} \tag{19}$$

Substituting the modified CAPM from Equation 13 into the WACC from Equation 18, we get the WACC for Company i:[24]

$$WACC_i = \left[r_f + \beta_i(r_m - r_f) + \gamma S_i\right] * E_i + k_i^d * (1 - x_i)D_i \tag{20}$$

Rearranging to highlight the size term, we have

$$WACC_i = \gamma S_i * E_i + \left[r_f + \beta_i(r_m - r_f)\right] * E_i + k_i^d * (1 - x_i)D_i \tag{21}$$

Substituting Equation 21 into Equation 19, we get the market value of firm i:

$$V_i^F = c_i^F * \frac{1}{\gamma S_i * E_i + \left(r_f + \beta_i(r_m - r_f)\right) * E_i + k_i^d * (1 - x_i)D_i - g_{c_i^F}} \tag{22}$$

The firm-level capitalization rate resulting from Equation 22 is then

$$z_i^F = \gamma S_i * E_i + \left(r_f + \beta_i(r_m - r_f)\right) * E_i + k_i^d * (1 - x_i)D_i - g_{c_i^F} \tag{23}$$

As before, this capitalization rate may be size-adjusted by replacing S_i with S_j, where $j \neq i$. This results in the firm-level capitalization rate for Company j,

$$z_j^F = \gamma S_j * E_j + \left(r_f + \beta_i(r_m - r_f)\right) * E_i + k_i^d * (1 - x_i)D_i - g_{c_i^F} \tag{24}$$

and the firm-level pricing multiple for Company j,

$$M_j^F = \frac{1}{z_j^F} \tag{25}$$

In practice, "replacing" S_i with S_j involves subtracting the size term from the capitalization rate and then adding back the corrected term. For pricing multiples involving cash flow to equity, the size term is just γS. For pricing multiples involving cash flow to the firm, the size term is $\gamma S * E$. Mechanically, this looks like Equation 26 for equity pricing multiples and Equation 27 for firm-level pricing multiples:

$$z_j^e = z_i^e - \gamma S_i + \gamma S_j \tag{26}$$

$$z_j^F = z_i^F - \gamma S_i * E_i + \gamma S_j * E_j \tag{27}$$

Illustrations using Equations 26 and 27 are provided in Section V.

IV. Capitalization Adjustment Methodology

We return to Equation 15 to describe the capitalization adjustment, which shifts focus to the systematic risk coefficient, β:

$$z_i^e = r_f + \beta_i^L(r_m - r_f) + \gamma S_i - g_{c_i^e} \tag{28}$$

Equation 28 adds the superscript L to the systematic risk coefficient (β_i^L) to reflect the fact that this beta is leveraged according to the capitalization of Company i. To adjust for capitalization differences between Company i and Company j, we start by de-levering β_i^L to get Company i's unlevered beta (β_i^U) using the practitioners' method described in Section II. Rewriting Equation 14 for Company i, we have

$$\beta_i^U = \frac{\beta_i^L}{1 + \frac{D_i}{E_i}}$$

D and E continue to refer to the debt capitalization and equity capitalization, respectively, of Company i. Once the unlevered beta is found, it may be re-levered using the capitalization of Company j:

$$\beta_j^L = \beta_i^U\left(1 + \frac{D_j}{E_j}\right) \tag{29}$$

With the newly levered beta reflecting Company j's capitalization, the equity-level capitalization rate from Equation 28 can be rewritten for Company j:

$$z_j^e = r_f + \beta_j^L(r_m - r_f) + \gamma S_i - g_{c_i^e} \tag{30}$$

24 For simplicity, the portion of the WACC attributable to preferred stock is not shown. Since the adjustment for size involves only the portion of WACC attributable to common equity, this exclusion does not change the spirit of the adjustment.

This capitalization rate is then inserted into Equation 9 to derive the capitalization-adjusted equity pricing multiple.[25]

As with the size adjustment, the capitalization adjustment must be altered for pricing multiples involving cash flows to the firm. The same procedure may be followed as before, resulting in capitalization terms $\beta^L (r_m - r_f)$ and $\beta^L (r_m - r_f) * E$ for equity-level and firm-level pricing multiples, respectively. Mechanically, this looks like Equation 31 for equity pricing multiples and Equation 32 for firm-level pricing multiples:

$$z_j^e = z_i^e - \beta_i^L(r_m - r_f) + \beta_j^L(r_m - r_f) \tag{31}$$

$$z_j^F = z_i^F - \beta_i^L(r_m - r_f) * E_i + \beta_j^L(r_m - r_f) * E_j \tag{32}$$

While no example of the capitalization adjustment is provided, the mechanics mimic those of the size adjustment shown in Section V.

V. Examples

To facilitate understanding of the pricing multiple adjustments applied in this article, we provide the following examples of adjustments for both equity- and firm-based pricing multiples. A mix of real-world and fictitious data is used to balance realism and simplicity; hence, no valuation date is given.

Equity-Level Size Adjustment

We start with two companies:

- Company i: Under Armour, Inc., a publicly traded apparel manufacturing company
- Company j: ABC Company, a private apparel manufacturing company we are valuing

The only equity-level pricing multiple we consider is the price-to-income ratio, where price is represented by market capitalization and income by net income. Here is the information we know about these two companies:

Under Armour:

- Market capitalization (V_i^e) = $7.0 billion
- Net income (c_i^e) = $525.0 million

ABC Company:

- Market capitalization (V_j^e) = *unknown*
- Net income (c_j^e) = $125,000

Our goal is to estimate ABC Company's market capitalization using Under Armour as the sole guideline public company. We start by restating the equity-level pricing multiple in the form of Equations 1 and 9:

$$M_i^e = \frac{V_i^e}{c_i^e} = \frac{1}{z_i^e}$$

Solving for Under Armour, we get

$$M_i^e = \frac{V_i^e}{c_i^e} = \frac{7.0\ billion}{525.0\ million} = 13.33$$

This is the unadjusted price multiple. To size-adjust it, we start by inverting this multiple to get the equity-level capitalization rate for Under Armour:

$$z_i^e = \frac{1}{M_i^e} = \frac{1}{13.33} = 0.075$$

Next, we adjust this capitalization rate for the difference in size between Under Armour and ABC Company. With a market capitalization of $7.0 billion, using Kroll's (formerly known as Duff & Phelps) CRSP[26] size premium by decile, Under Armour falls into the fourth decile, suggesting a size premium (γS_i) of 0.54 percent. While we do not yet know the market capitalization for ABC Company, we start by assuming that it will be in the 10th decile, suggesting a size premium (γS_j) of 4.8 percent.

The size adjustment for an equity-level pricing multiple was derived in Equation 26:

$$z_j^e = z_i^e - \gamma S_i + \gamma S_j$$

Substituting the values above gives us

$$z_i^e = 0.075 - 0.0054 + 0.048 = 0.1176$$

Inverting z_j^e, consistent with Equation 17, the size-adjusted price-to-income ratio for ABC Company is

$$M_j^e = \frac{1}{z_j^e} = \frac{1}{0.1176} = 8.50$$

25 Under the sum beta methodology described in Section IV, the capitalization adjustment is applied to the total sum beta, which is then decomposed into the concurrent and lagged betas using the same proportions of total beta prior to the capitalization adjustment.

26 CRSP data is provided by the Center for Research in Security Prices.

Using this size-adjusted price-to-income ratio, we estimate ABC Company's market capitalization, rounded to the nearest thousand, to be

$$V_j^e = c_j^e M_j^e = 125{,}000 * 8.50 = 1{,}063{,}000$$

As you can see, the estimated market capitalization of $1,063,000 for ABC Company falls closest to the CRSP 10th decile, implying that we selected the size premium appropriately. If, after estimating ABC Company's market capitalization, we find it to fall into a different decile than the one we assumed, we would repeat the exercise, choosing the size premium from the more appropriate decile.

Firm-level Size Adjustment

We start with the same two companies from the previous equity-level size adjustment example. The firm-level pricing multiple we consider is the enterprise-value-to-EBITDA ratio. Here is the information we know about these two companies:

Under Armour, Inc.:

- Enterprise value (V_i^e) = $7.5 billion
- EBITDA (c_i^e) = $650.0 million
- Equity capitalization (E_i) = 85 percent

ABC Company:

- Enterprise value (V_j^e) = Unknown
- EBITDA (c_j^e) = $145,000
- Equity capitalization (E_j) = 95 percent

Our goal is to estimate ABC Company's enterprise value using Under Armour as the sole guideline public company. We start by stating the firm-level pricing multiple as

$$M_i^F = \frac{V_i^F}{c_i^F} = \frac{1}{z_i^F}$$

Solving for Under Armour, we get

$$M_i^F = \frac{V_i^F}{c_i^F} = \frac{7.5\ billion}{650.0\ million} = 11.54$$

Once again, this is the unadjusted price multiple. To size-adjust it, we start by inverting this multiple to get the firm-level capitalization rate for Under Armour:

$$z_i^F = \frac{1}{M_i^F} = \frac{1}{11.54} = 0.0867$$

Next, we adjust this capitalization rate for the difference in size between Under Armour and ABC Company. With a market capitalization of $7.0 billion, using Kroll's CRSP size premium by decile, Under Armour falls into the fourth decile, suggesting a size premium (γS_i) of 0.54 percent. While we do not yet know the market capitalization for ABC Company, we start by assuming that it will be in the 10th decile, suggesting a size premium (γS_j) of 4.8 percent.

The size adjustment for a firm-level pricing multiple was derived in Equation 27:

$$z_j^F = z_i^F - \gamma S_i * E_i + \gamma S_j * E_j$$

Substituting the values above gives us

$$z_j^F = 0.0867 - 0.0054 * 0.85 + 0.048 * 0.95 = 0.1277$$

Inverting z_j^F, consistent with Equation 25, the size-adjusted enterprise-value-to-EBITDA ratio for ABC Company is

$$M_j^F = \frac{1}{z_j^F} = \frac{1}{0.1277} = 7.83$$

Using this size-adjusted enterprise-value-to-EBITDA ratio, we estimate ABC Company's enterprise value, rounded to the nearest thousand, to be

$$V_j^F = c_j^F M_j^F = 145{,}000 * 7.83 = 1{,}135{,}000$$

When deriving firm-level value, one should convert back to market capitalization[27] to once again ensure that the most appropriate size decile was selected. As before, if, after estimating ABC Company's market capitalization, we find it to fall into a different decile than the one we assumed, we would repeat the exercise choosing the size premium from the more appropriate decile. Additionally, while the equity capitalization is given in this example, it is also an iterative input, since changes in enterprise value imply changes in equity capitalization. Should completion of this exercise show the assumed equity capitalization to be materially different, the exercise should be repeated with a more accurate equity capitalization.

27 Market capitalization = enterprise value – net debt.

VI. Conclusion

This article has explored the literature surrounding a risk premium for size differences, concluding that adjustments to pricing multiples for both size and capitalization are reasonable and supported empirically. An approach was described for adjusting publicly observable equity- and firm-level pricing multiples to account for differences in size and capitalization between two different firms. Adjustments are presented in a practical way, allowing practitioners to easily apply the defined methods. These methods have particular use in the valuation of privately held companies; specifically with respect to application of the GPCM.

It will be necessary for practitioners to obtain proprietary or externally produced estimates of size premiums. Traditionally, many practitioners have relied on valuation handbooks[28] published by Kroll (formerly known as Duff & Phelps) for size premium data. In this source, size premiums are presented by decile, with the smallest (10th) decile further decomposed into quarters. Practitioners may place all companies from their guideline group into their corresponding deciles to recover the size premium embedded in all observable pricing multiples. These size premiums are then replaced with that of the privately held company, similarly placed into its corresponding decile.

Following the methods described in this article will produce pricing multiples that have already considered size and capitalization in an objective way. Subjective judgment may still be necessary in selecting the appropriate pricing multiple from the guideline group to apply to the privately held company, though less so than without making such adjustments for size and capitalization. VE

Derek Zweig, CFA, FRM, cofounded Value Analytics LLC in 2020 and currently serves as its chief executive officer. Mr. Zweig is an expert in asset valuation and has performed or contributed to over 1,000 valuation engagements involving businesses, business interests, and intangible assets of privately held and publicly traded companies. He is an active member of the CFA Institute and the Global Association of Risk Professionals. Mr. Zweig earned an MS in applied economics from Johns Hopkins University and a BS in finance from The Ohio State University. Email: derek@valueanalytics.org.

Adam Luke, PhD, cofounded Value Analytics LLC as chief technology officer. Before founding Value Analytics, Mr. Luke worked as a data scientist at both established and early phase technology companies. His experience across business scales and application domains provides a robust background for producing innovative software services. Mr. Luke holds a PhD in engineering from the University of California, Irvine. Email: adam@valueanalytics.org.

28 The valuation handbooks have been replaced by Kroll's Cost of Capital Navigator. For example, all of the data, information, and content from the *Valuation Handbook—U.S. Guide to Cost of Capital* is now available in the Cost of Capital Navigator's U.S. Cost of Capital Module. *See* https://www.kroll.com/en/cost-of-capital/frequently-asked-questions.

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