

Business Valuation Review™

Volume 36
Issue 1
Spring 2017

- 1 Editor's Column
Dan McConaughy, PhD, ASA
- 3 American Society of Appraisers Business
Valuation Committee Special Topics Paper #3
The Use of Management's Prospective Financial
Information by a Valuation Analyst
- 7 Petroleum Property Income and Market Valuation
Approaches (Transactions Beware!)
Louis R. Posgate, ASA, PE, CMA
- 9 A Primer on Bargain Purchases and Negative Goodwill
Dan Daitchman, ASA
- 15 Statutory Fair Value in Dissenting Shareholder Cases—
Part I
Gilbert E. Matthews, MBA, CFA
- 32 Comparing Three Convertible Debt Valuation Models
Dwight Grant, PhD
- 42 From the Chair of the Business Valuation Committee
William A. Johnston, ASA

BV
Review

ASA
American Society of Appraisers
Providing Value Worldwide



BVR™ Editorial Review Board

Editor

Daniel L. McConaughy, PhD, ASA
California State University Northridge

Managing Editor

Rita Janssen
Lawrence, Kansas

Business Manager

James O. Brown, ASA
Campbell, California

Reviewers

Ashok Abbott*, PhD
Morgantown, West Virginia

John D. Finnerty, PhD
New York, New York

Stillian Ghaidarov

Roger Grabowski*, FASA
Chicago, Illinois

*Editorial Board Member

Editors Emeritus

James H. Schilt, ASA (1982–2001)
San Francisco, California

Jay E. Fishman, FASA (2002–2007)
Bala Cynwyd, Pennsylvania

Roger J. Grabowski, FASA (2008–2012)
Chicago, Illinois

Publishers Emeritus

John E. Bakken, FASA (1982–2003)
Granby, Colorado

Ralph Arnold, III, ASA (2004–2007)
Portland, Oregon

Editor's Column

Dan McConaughy, PhD, ASA

Hello Everyone,

Recently, I reviewed a valuation report with a sensitivity analysis matrix. Such a matrix is a common feature in valuation reports.

This time I looked at it a little differently, thinking about the relationship between the expected cost of capital and expected growth. The traditional interpretation of this matrix, taken from the valuation and edited to disguise the real numbers and shown below, gives the range as the best case to worst case range, where best case

is high growth and low discount rate, and worst case is high discount rate and low growth.

The Gordon Growth Model suggests that r and g are related: $r = (D1 / P) + g$. If they are related and the Gordon Growth Model reflects an expectation, would a more reasonable range be read along the diagonal from lower right to upper left?

Please let me know your thoughts.

Dan McConaughy

Sensitivity Analysis						
Discount Rate		Long term growth rate				
		3.0%	3.5%	4.0%	4.5%	5.0%
18.0%		430.0	441.8	454.6	468.1	482.9
19.0%		402.0	412.2	423.1	434.8	447.2
20.0%		377.4	386.3	395.8	405.7	416.5
21.0%		355.6	363.2	371.5	380.3	389.5
22.0%		336.0	342.8	350.0	357.7	365.9
Enterprise Value Range		\$ 363.2	-	\$ 434.8		
More Reasonable Range?		\$380.3	-	\$412.2		

$$r = (D1 / P) + g$$

American Society of Appraisers Business Valuation Committee Special Topics Paper #3 The Use of Management's Prospective Financial Information by a Valuation Analyst^a

According to AICPA Professional Standards: AT Section 301 Financial Forecasts and Projections, “financial forecast is the prospective financial statements that present, to the best of the responsible party’s knowledge and belief, an entity’s expected financial position, results of operations, and cash flows. A financial forecast is based on the responsible party’s assumptions reflecting the conditions it expects to exist and the course of action it expects to take.” In order for a valuation analyst to objectively perform the valuation analysis, the analyst has to judge whether or not management’s prospective financial information is reasonable and can be relied upon in the valuation analysis. This white paper will focus on the valuation analyst’s role in using management’s forecast financial information and suggest a few useful analytical tools available to the valuation analyst.

Introduction

Business valuation is the process of determining the economic value of a business entity, which is based on the ability of the business to generate future cash flows. One common method used in estimating the value of the entity, the discounted cash flow (DCF) method, utilizes free cash flow expected in the future and discounts those prospective cash flows at a risk-adjusted rate to arrive at a net present value of or for the business. The DCF method is particularly useful when future profit margins and growth are expected to vary significantly from historical operating results.

The two common components of the DCF method are:

- an estimate of future cash flows, and
- an estimate of an appropriate risk-adjusted required rate of return used to discount the estimated future cash flows back to net present value.

The credibility of the DCF method lies in both a reliable forecast and a well-developed discount rate. Much has already been written about discount rate development. This white paper will focus on the valuation analyst’s role in using management’s forecast financial information.

^aThis white paper is for education purposes and should not be considered as authoritative. It has been provided for discussion of a concept and is not being offered as professional advice. Each set of circumstances may require a different analysis to be performed.

Management’s Prospective Financial Information (PFI): Difference between a Forecast and a Projection

Even though forecast and projection are used interchangeably by valuation analysts, they are actually different concepts in accounting literature.

According to AICPA, “financial forecast is the prospective financial statements that present, to the best of the responsible party’s knowledge and belief, an entity’s expected financial position, results of operations, and cash flows. A financial forecast is based on the responsible party’s assumptions reflecting the conditions it expects to exist and the course of action it expects to take.”¹

“Financial projection is the prospective financial statements that present, to the best of the responsible party’s knowledge and belief, given one or more hypothetical assumptions, an entity’s expected financial position, results of operations, and cash flows. A financial projection is sometimes prepared to present one or more hypothetical courses of action for evaluation, as in response to a question such as ‘What would happen if...?’ A financial projection is based on the responsible party’s assumptions reflecting conditions it expects would exist and the course of action it expects would be taken, given one or more hypothetical assumptions.”²

¹American Institute of Certified Public Accountants, AICPA Professional Standards, “AT Section 301 Financial Forecasts and Projections.”
²Ibid.

Based upon AICPA guidance, a forecast is what is expected to happen, whereas a projection is what might happen given certain hypothetical assumptions. For example, if the subject company has a mature and stable business, its historical performance may be a good indicator of its future performance. Management's PFI is a forecast because there are no hypothetical assumptions. If the subject company is an early-stage company without any revenue, its future performance depends on a variety of factors, such as key patent approval, a new round of financing, and success of its marketing plan, in which case management's PFI may be a projection because the PFI relies on various hypothetical assumptions. The valuation analyst should understand the distinction between a forecast and projection.

Professional Standards on PFI

To understand the valuation analyst's role in using management's PFI, it is important to understand the ASA's position on this issue in valuation professional standards. Standard BVS-VIII: Comprehensive Written Business Valuation Report broadly specifies that, "If projections of balance sheets or income statements are used in the valuation, key assumptions underlying those projections must be included and discussed."³

USPAP does not provide specific guidance on use of management's PFI in a business valuation. However, in Statement on Appraisal Standards No. 2 (Real Property), USPAP states:

"To avoid misuse or misunderstanding when DCF analysis is used in an appraisal assignment to develop an opinion of market value, it is the responsibility of the appraiser to ensure that the controlling input is consistent with market evidence and prevailing market attitudes. Market value DCF analyses should be supported by market derived data, and the assumptions should be both market and property specific. Market value DCF analyses, along with available factual data, are intended to reflect the expectations and perceptions of market participants. They should be judged on the support for the forecasts that existed when made, not on whether specific items in the forecasts are realized at a later date."⁴

In general, a valuation analyst (in a broad sense) should make a determination of whether or not the PFI prepared by management is reasonable for use in performing a valuation.

Assessing this information for reasonableness means that a valuation analyst should not simply accept management's PFI without understanding the assump-

tions made in the PFI. The ASA's professional standards suggest that the valuation analyst should understand the nature of management's forecast and the underlying assumptions and discuss them in the valuation report. It is good practice for the valuation analyst to understand how and why the PFI is prepared and determine the reasonableness of the assumptions. Understanding the assumptions behind management's PFI increases the credibility and reliability of the valuation.

Role of Valuation Analyst

In order for a valuation analyst to objectively perform the valuation analysis, the analyst has to judge whether or not management's PFI is reasonable and can be relied upon in the valuation analysis. The first step in understanding management's assumptions is determining whether the PFI was prepared using a top-down approach or a bottom-up approach.

A top-down PFI starts with a business assessing the market as a whole. First, management estimates the current market size available for their business and factors in relevant sales trends. From that, management can then identify their own company's target sales. The assumption is that, given the existing market and potential market growth, the company can expect to capture a certain percentage share of the market in subsequent years.

Conversely, a bottom-up PFI is a detailed budget typically developed from spending plans by various groups within the company. The bottom-up approach is grounded in the product or service itself, from which a PFI is made based on what the company needs to get its offering to the market (i.e., things like how many employees the company has, how many factories it can open, or how many clients it can attract). Also known as an operating expense plan, bottom-up PFI examines factors such as production capacity, department-specific expenses, and addressable market in order to create a more accurate sales forecast.

While it is clear that both top-down and bottom-up forecasting techniques have their advantages, the best model may ultimately depend on the nature of the specific business. Firms that experience little deviation in cash flow from one month to the next may benefit from a top-down PFI model. Additionally, top-down models can be effective for startups that do not have any accumulated sales data. On the other hand, bottom-up forecasting may be ideal for a seasonal business that experiences significant variation in cash flows throughout the year.

In summary, top-down models start with the entire market and work down, while bottom-up PFIs begin with the individual business department and expand out. Understanding the pros and cons of both types of financial forecasting is important for the valuation analyst

³American Society of Appraisers, ASA Business Valuation Standards.

⁴The appraisal foundation Uniform Standards of Professional Appraisal Practice.

so they can assess the reasonableness and credibility of management's PFI.

In addition to understanding the approach management used to develop the PFI, a valuation analyst should understand who actually prepared the PFI to consider any potential biases. For example, if the marketing and sales department prepared the PFI, is it potentially too optimistic? Conversely, if the finance and accounting department prepared the PFI, is it too pessimistic? Typically, the greater amount of time and company personnel dedicated to the forecast planning process, the more detailed and accurate the forecast will be. Additionally, if the valuation analyst has access to previous forecasts prepared by management, the analyst should compare the previous forecasts to actual results to consider the accuracy of management with those PFI reports.

The valuation analyst should also understand the fundamental assumptions that drive the forecast. If the assumptions in the PFI prepared by management are not readily apparent, the valuation analyst may consider asking questions of management regarding the underlying assumptions.

Examples of questions that the valuation analyst may consider in understanding the assumptions behind the PFI include:

- Is expected growth in revenue due to an increase in price or volume or both?
- How does expected growth in revenue of the company compare to industry growth?
- Is revenue growth achievable given the current conditions of company operations?
- How are new products or services considered in forecasted revenue? If so, are corresponding expenses reasonable?
- Are new products under development? What is the basis for research and development expenses? Are forecasted capital expenditures consistent with the revenue growth assumptions?
- Are operating expenses consistent with historical levels? Did management differentiate between fixed and variable costs?
- If there are variable costs, what do costs vary against?
- Are forecasted results consistent with historical results? If not, why?
- In a business combination, do the forecasts consider any synergies from revenue enhancement and/or cost savings?
- Is it reasonable for management to forecast a much higher or lower growth rate compared to guideline companies or other industry metrics?

- Is it reasonable for management to forecast a much higher or lower profit margin compared to guideline companies or other industry metrics?

A good benchmark with which to evaluate the reasonableness of management's PFI is industry data. The valuation analyst should compare the subject company's historical performance and management's PFI to those of the guideline publicly traded companies. Comparison with guideline publicly traded companies can also provide the valuation analyst with detailed industry information, such as normalized working capital level, average industry growth rate, and average capital expenditures. In addition, the valuation analyst might research market and industry research reports and relevant government data as additional information to determine whether or not the PFI is reasonable for use in a valuation.

Furthermore, when utilizing more than one approach in valuation, if the valuation results from use of the PFI are significantly different from other valuation methods, it may be good practice to reevaluate the reasonableness of management's PFI.

What if Management Doesn't Prepare a PFI?

In certain circumstances, management may not prepare a PFI. In those circumstances, a PFI may be available from other sources, such as the company's outside financial advisors. If the subject company is a public company, equity analysts may prepare prospective information in research reports. The valuation analyst should consider reconciling multiple sources of PFI in preparing for valuation analysis.

Occasionally, there is no PFI available to valuation analyst. This is particularly common when dealing with small, privately held companies. In these circumstances, the valuation analyst may ask management to develop the PFI specifically for the valuation. In such case, management still ultimately takes responsibility for the PFI.

If management prepares a PFI, but a valuation analyst finds management's PFI to be unreasonable, the valuation analyst should first make recommendations for any revisions in the PFI to management. Otherwise, the valuation analyst should consider any additional risk in achieving the PFI company-specific risk premium in the cost of capital to perhaps capture any additional forecast risk. If the valuation analyst finds that management's PFI is not reasonable for use in the valuation, the analyst may also consider using only valuation methods that do not require the use of PFI. In these situations, a valuation analyst may consider making clear in the analysis the limitations of the data available for the analysis. In

extreme circumstances, the valuation analyst should consider resigning from the engagement due to lack of appropriate data.

Finally, it may be good practice for the valuation analyst to have representations that the PFI provided to the valuation analyst is management's best estimate of expected future performance of the company through a management representation letter. Management representation letters are commonly used by third-party auditing firms in their financial reporting engagements.

Analytical Tools

Whether the goal is to evaluate the reasonableness of management's PFI or to assist management to prepare for a PFI, there are a few useful analytical tools available to the valuation analyst. It is common for historical results to be used as a starting point in any forecast. One way to statistically evaluate PFI is for the analyst to perform a regression analysis comparing the company's historical financial performance to certain economic indicators. One key to this approach is to identify the salient economic variable(s) that correlate with historical revenue. Once the trending line is fitted, it is much easier to forecast for the future.

Some business valuation analysts perform scenario analysis or sensitivity analysis when evaluating management's PFI. The analyst may consider utilizing three scenarios, such as best, worst, and neutral, representing management's optimistic, pessimistic, and status-quo outlook for the company to identify key value drivers in the PFI. Sensitivity analysis on specific assumptions is

another powerful tool that may be used by valuation analysts to understand the reasonableness of management's PFI.

Powerful analytical tools such as Monte Carlo simulation may be helpful in determining the reasonableness in using management's PFI in the valuation. Traditional DCF analysis is based upon single-point estimates of the variables. Monte Carlo simulation produces distributions of possible outcome values based upon distributions of underlying variables. Monte Carlo simulations calculate thousands of scenarios having different combinations of inputs. The simulation captures numerous "what-if" scenarios related to the company's future performance.

Summary and Conclusion

The valuation analyst should determine that any PFI prepared by management is reasonable for use in a valuation analysis. Assessing for reasonableness means that the valuation analyst should not simply accept management's PFI without understanding the assumptions made in the PFI. The ASA's professional standards suggest that the valuation analyst should understand the nature of management's forecast and the underlying assumptions and discuss them in the valuation report. Good practice dictates that the valuation analyst understand how and why the PFI was prepared and determine the reasonableness of the assumptions. Assessing reasonableness can take many forms, But determining the reasonableness of management's PFI increases the credibility and reliability of the valuation.

Petroleum Property Income and Market Valuation Approaches (Transactions Beware!)

Louis R. Posgate, ASA, PE, CMA

Characteristics exhibited by producing liquid-rich shale formations often cause inaccurate forecasts of natural gas and oil production when multistage hydraulic fracking is used for extraction. Poor cash flow estimates can be avoided by practicing due diligence when appraising petroleum property value and income. Due diligence is satisfied by comparing volumes recorded on actual royalty check stubs or monthly statements to state-reported production volumes, by relying on petroleum reserve appraisals instead of transaction multipliers or rules of thumb, and by using nearby analog wells in proximity found in subscription databases for good “type” well decline curves. In addition, properly referencing the contributing work of other appraisers, maps, and analyses improves understanding and reduces errors.

Introduction

A band of Eagle Ford Shale fields is being developed (e.g., Sugarkane [Eagle Ford] and Eagleville [Eagle Ford-1, Eagle Ford-2]) in quality regimes or “tiers” near San Antonio, Texas. Producing counties, such as Atascosa, Live Oak, Lavaca, Gonzales, La Salle, and contiguous or nearby counties through Fayette and Lee (west of Houston), contain tracts where wells were drilled vertically through untapped shales, rich in liquids, to targeted chalk and sandstone traps. However, relatively recently, wells have been drilled horizontally and snake through the shale beds. Shale is composed of overburden-compressed hydrocarbon-imbedded clays that are the source rock providing the hydrocarbons. The hydrocarbons migrate through natural and hydraulically induced fractures to conventionally completed sandstone sedimentary and carbonate reef-like reservoirs. Connected (permeable) pores in such reservoirs are produced by solution gas depletion. An example would be shaking up canned soda.

Over a decade ago in the Newark East Barnett Shale Field (a Mississippian-aged, 8,000 ft deep shale) south and west of Dallas, expansion of multistaged hydraulic

fracking through horizontal drainage was performed by Mitchell Energy and other operators with longer (now up to 2 mi long) horizontal, sometimes multilateral bores, thereby increasing the effective shale thicknesses from 100 ft (30 meters vertically) to many thousands of feet. Under pressure, oil flushes through fractures in shale, then seeps through the shale walls that have 1/1000th of the permeability of sandstones.

This causes the production (and cash flow profile) to decline hyperbolically (initially steeply and then stabilizing at a lower rate, often five to six years or more later, exponentially (a straight line on a semilog plot) like conventional reservoirs. *This makes the oil and gas price assumptions for the first two years critical to an appraiser’s reserves and present values, drilling, and royalty property economics.*

Specific factors, such as if the wells are “choked back” (to flatten the decline; e.g., like holding a thumb over a shaken soda can), if petroleum vapors that condense are captured (e.g., by natural gas liquids extraction plants downstream paying owners for their share of extracted product), and if the length of laterals and sand/proppant treatment (millions of lbs) and number of gallons (hundreds of thousands) in the frack effectively prop open fractures to facilitate flow, point to the importance of using analog wells in close proximity for good “type” well decline curves. These data may be found in subscription databases used by petroleum engineers/appraisers.

Sometimes unstable pressure declines in an early stage of production, with higher gathering system pressure and high (62+) condensate gravity (API number indicating volatility), can pose a production prediction error. This is

Louis R. Posgate is an Accredited Senior Appraiser, designated in both business valuation and oil and gas valuation, by the American Society of Appraisers. He is a licensed professional engineer and certified minerals appraiser (CMA), and a valuation consultant in Driftwood, Texas, d.b.a. LRP Business Appraisal, an affiliate of Mineral Valuation Specialists.

due to gas and oil volume shrinkage—I have seen up to 40% volume shrinkage from reported production to volumes on which owners are paid.

Statistical regression techniques applied by analysts *without* reviewing actual purchased production for shrinkage, shut-in wells being worked on, or drilling well buildup in a lease can indicate a different *prospective* field decline rate and can cause valuation inaccuracy. In Texas, a pooled unit can have several completed wells constituting a *lease*, often with *planned* “PUD” additions to be included. Since high well-stream pressures in gassy oil- or liquids-rich reservoir windows can cause significant condensate and gas volume shrinkage from wellhead-measured (state-reported) volumes, cash flow streams evaluation software must reflect this adjustment. Comparison of state-reported volumes to actual royalty check stub volumes is required to estimate not only shrinkage, but 2× to 3× the gas equivalent price when natural gas liquid revenues are included. This practitioner has dealt with this in producing fields in Live Oak, Gonzales, Atascosa, and other Texas counties hosting Eagle Ford Shale fields.

One problem regarding business valuers using transaction multiples as a proxy for reserves appraisal is not using volume adjustments, i.e., gas BTU equivalence with one barrel oil. This is magnified if the thumb rule for dollars per equivalent barrel of daily production multiple is relied on to avoid reserves or royalty appraisal costs, without the necessary BTU or similar equivalence adjustments. Usually nothing other than a good petroleum appraisal is acceptable for users valuing companies, partnerships, and business components. This appraiser has observed this misuse of market multiples, assuming \$/BTU-equivalence and similar volume shrinkage, when thumb rules are applied for valuing oil and gas reserves, and questionably comparable acquisition multiples are used in such a transaction method. This was noted in energy papers presented in ASA conference speeches, and by reviewing court case decisions (analysis in “Oil

and Gas Federal Tax Case Review: Fair Market Values with Volatility,” *Business Valuation Review* 21, 4 [December 2002, pp. 181–185]), and in articles on web sites.

Due diligence is practiced by comparing royalty statement monthly volumes to state-reported production, asking about the subject’s oil API gravity and if any shrinkage exists, and then reducing, if necessary, production volumes by shrinkage when fitting data to decline curves. This is significant in *volatile*-rich gas windows of the Eagle Ford Shale, and any other volatile crude and condensate nonconventional shales. Without the petroleum appraisals, transfers such as estate asset values or capital gains basis, sometimes being estimated retrospectively, could cause a client to have a costly challenge or misstatement of fair market value, or an unreasonable acquisition purchase price of reserves, royalty property, or asset value. This satisfies the competency requirement under USPAP.

Other competency and ethics items to consider involve properly referencing appraisers who contribute (*significantly*) to opinions stated in the report. When such an opinion of value is formed by reference in part to any other appraiser’s work or research, its provider or source should be listed in the report references. That applies to persons doing any contributing work subcontracted for an appraisal, such as running production decline curves for reserves estimation, doing market analysis, calculating discount rate of company stock versus asset valuations, sharing maps or privately developed data, and referencing papers or presentations. When quoting papers or footnoting, such as another appraiser’s published document on lease bonus multiples applied to nonproducing acreage, a mineral parcel, or lease transaction comparability, etc., the referencing author should contact the original appraiser to understand the original context and whether it is appropriate to apply it to the current valuation. This applies when citing it in published valuation documents, magazines, etc., to avoid errors.

A Primer on Bargain Purchases and Negative Goodwill

Dan Daitchman, ASA

When a change of company control occurs, such as an acquisition, a valuation of the assets acquired must be performed to be compliant with generally accepted accounting principles, as mandated by the Financial Accounting Standards Board (FASB) and addressed in Accounting Standards Codification (ASC) 805: Business Combinations. This type of exercise is commonly referred to as a purchase price allocation, since the purchase price of the subject company is allocated across all tangible and intangible assets and liabilities acquired. Generally, the value of the subject company is greater than the value of the acquired assets, or in other words, “the whole is greater than the sum of the parts.” However, what if the sum of the parts is greater than the whole? This paper looks at transactions involving fair value and bargain purchases, the differences between the two, and how bargain purchases should be addressed.

Background

When a change of company control occurs, such as an acquisition, a valuation of the assets acquired must be performed to be compliant with generally accepted accounting principles, as mandated by the Financial Accounting Standards Board (FASB) and addressed in Accounting Standards Codification (ASC) 805: Business Combinations.¹ This type of exercise is commonly referred to as a purchase price allocation, since the purchase price of the subject company is allocated across all tangible and intangible assets and liabilities acquired. Generally, the value of the subject company is greater than the value of the acquired assets, or in other words, “the whole is greater than the sum of the parts.” That additional value is referred to as goodwill. However, what if the sum of the parts is greater than the whole? There are certain transactions in which the total value of the individual assets acquired in a

transaction exceeds the price paid for the total company. This is often referred to as a “bargain purchase.” The scope of this paper is to look at transactions involving fair value and bargain purchases, the differences between the two, and how bargain purchases should be addressed.

Transactions with Positive Goodwill

In a typical acquisition, acquired tangible assets may include working capital (accounts receivable, inventory, etc.), personal property (machinery and equipment), and real property. In addition, there are a number of intangible assets that are often acquired, which are seen as the “value-drivers” of the company. An asset must pass one of two tests to have allocated value: (a) It must be of a legal or contractual nature; or (b) it must be separable from the business. Such intangible assets can include a brand name, patented or unpatented technology, certifications or licenses, noncompetition agreements, customer relationships, as well as industry-specific forms of intangible assets, such as broadcast licenses or distribution rights.

In this exercise, after allocating value to all of these assets, the residual difference between the fair value of the acquired assets and liabilities versus the purchase price is considered goodwill. Hence, the aggregate fair value of the acquired assets plus the fair value of goodwill equals the purchase price of the transaction, assuming the overall transaction itself is conducted at fair value.

Dan Daitchman is a manager with Great American Group in their Corporate Valuation Services practice. Dan provides valuations of assets, including business interests, intellectual property, and various other intangible assets. His valuation work is primarily used for financial reporting, tax, asset-based lending, transaction advisory, as well as fairness and solvency opinions. Dan is also an Accredited Senior Appraiser of the American Society of Appraisers. He can be reached at ddaitchman@greatamerican.com.

¹ASC 805: Business Combinations.

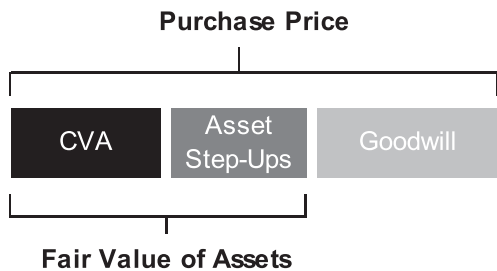


Figure 1
Purchase Price Diagram

Purchase accounting requires the use of the standard of fair value, which is defined in ASC 820² as: "... the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date."

An important concept of this definition is that of a "market participant," which is an actual or theoretical potential buyer of the business or asset (or assumer of the liability). ASC 805 does not require that specific market participants be identified, only that their characteristics be described.

The difference between the carrying values of the assets (CVA) acquired and the fair values of those assets represents a step-up in the basis for the acquired assets. A diagram showing CVA, the step-up from CVA to fair value, and goodwill as components of the purchase price is shown in Figure 1.

Goodwill is an asset representing the future economic benefits arising from other assets acquired in a business combination that are not individually identified and separately recognized.³ There are many ways to more explicitly describe what comprises goodwill, but often it includes the acquired company's workforce, prospects for future growth, market participant synergies with the target company, and "going concern value" from the assemblage of the assets. Table 1 shows a sample of what a purchase price allocation might look like for a \$60 million acquisition.

As you can see, the fair value of the acquired assets sums up to \$42.5 million, which means that the residual \$17.5 million represents goodwill. It is also important to note that while the trained and assembled workforce (TAW) is listed separately, it is not viewed as separable from the company and hence is bundled into goodwill on the subject company's balance sheet. For purchase price allocation purposes, it is listed separately because it is used as an input in valuing intangible assets under the

²ASC 820-10-20: Fair Value Measurements and Disclosures.

³ASC 805-10-20.

Table 1

Purchase Price Allocation with Positive Goodwill (\$ in 000s)

Assets	Fair Value	% of Total
Net Working Capital	\$15,000	25.0
Personal Property	10,000	16.7
Real Property	1,000	1.7
Identified Intangible Assets		
Patents and Technology	5,000	8.3
Trade Name	7,000	11.7
Customer Relationships	4,000	6.7
Unallocated Intangible Assets		
Trained and Assembled Workforce	500	0.8
Goodwill	17,500	29.2
Purchase Price	\$60,000	100.0

Multi-Period Excess Earnings Method (MPEEM). This valuation method will be discussed later in the paper.

Transactions with Negative Goodwill

While most transactions have positive goodwill, occasionally the fair value of the acquired assets exceeds the purchase price. This scenario results in a nontaxable gain and is commonly referred to as a bargain purchase. Using the previous example, if the purchase price were only \$40 million instead of \$60 million, Table 2 shows how the allocation would look. This type of scenario creates a significant amount of additional analysis, which will be described in further detail.

Table 2

Purchase Price Allocation with Negative Goodwill (\$ in 000s)

Assets	Fair Value	% of Total
Net Working Capital	\$15,000	37.5
Personal Property	10,000	25.0
Real Property	1,000	2.5
Identified Intangible Assets		
Patents and Technology	5,000	12.5
Trade Name	7,000	17.5
Customer Relationships	4,000	10.0
Unallocated Intangible Assets		
Trained and Assembled Workforce	500	1.3
Fair Value of Identified Assets	\$42,500	106.3
Goodwill	(2,500)	-6.3
Purchase Price	\$40,000	100.0

Characteristics—Initial Screening

There are several telltale signs that a transaction may be a bargain purchase. Two common themes are company distress and information asymmetry. Some earmarks of bargain purchases include:

- The seller was compelled to sell the business.
- The subject company has incurred financial losses in recent years.
- The transaction was not well marketed (i.e., a limited number of potential buyers were contacted).
- There was only one bid for the subject company.
- The subject company was acquired over a very short time frame.
- There was information asymmetry, in which the buyer knew more about the future prospects of the business than the seller.
- The net book value of the acquired assets exceeded the purchase price.

As with any purchase price allocation, there should be a detailed explanation associated with the transaction as to why it was a bargain purchase, and steps should be taken to document why the purchase price is not representative of fair value. If you cannot clearly articulate why the purchase price allocation represents a bargain purchase, you may need to revalue each asset, or conclude that the fair value of the overall business is more than the purchase price (i.e., the transaction did not occur at fair value). In that case, the concluded fair value is the amount allocated to the acquired assets, and the excess of the fair value of the business above the purchase price would be recorded as an extraordinary gain.

Valuation Characteristics

Since a bargain purchase often implies that the subject company has endured some form of financial distress, the fair value of the acquired assets is often depressed as well. A few examples are stated below.

Inventory

Occasionally, the fair value of the subject company's inventory will be required, particularly if inventory comprises a large portion of the company's assets. If the fair value of the inventory is greater than its net book value (NBV), it is referred to as an "inventory step-up." In a bargain purchase scenario, however, the fair value of the inventory could be lower than its NBV, which would be referred to as an inventory step-down. For example, the inventory's fair value might be less than its NBV due to low gross margins, low turnover, product

obsolescence, raw material price changes, or high fixed costs.

Fixed assets

One common characteristic of a company acquired under a bargain purchase is that the company has excess, underutilized, or nonoperating fixed assets on the balance sheet. In such a case, it can be inferred that the cash flows of the subject company do not support the fair values of the fixed assets as a going concern. This situation can be rectified by applying economic obsolescence to the fixed assets, thereby writing the assets down to create a fair value in which the cash flows of the business do support the fixed assets. One must be aware of both the macro- and micro-impacts of applying this analysis. Applying too much economic obsolescence to fixed assets can push a bargain purchase into a positive goodwill scenario. When applying economic obsolescence to fixed assets, typically the lower-end constraint is orderly liquidation value, also known as value in-exchange. The case for economic obsolescence is that there are not enough cash flows to support ownership of the fixed assets. Therefore, it would be self-contradicting to state that there is insufficient cash flow to support the full value of the fixed assets in-use, and yet also conclude that there is cash flow beyond that generated by the aggregate assets. In other words, economic obsolescence and residual goodwill do not typically both exist in the same operating unit in a transaction setting.

Customer relationships

One intangible asset that is present in nearly every purchase price allocation is customer relationships. As one may infer, this asset represents the value of the company's power to generate future sales from its existing customers. This asset is often valued using the MPEEM. The MPEEM attempts to isolate the value of the customer relationships by assuming a scenario in which the buyer would only own the customer relationships, and all other assets (i.e., working capital, fixed assets, trade name, etc.) are rented. The cash flows of the business are therefore reduced by economic rents, or "contributory asset charges," which are the theoretical costs of renting all of the other assets. The excess cash flow that remains is referred to as the "excess earnings," which are then discounted to calculate the fair value of customer relationships.

Another characteristic often found in bargain purchases is that, since the company frequently does not generate enough cash flows to support the assets of the business, the contributory asset charges applied in the MPEEM

Table 3
IRR Using the Market Participant Cash Flows and the Purchase Price

	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025
DFNCF	\$6,939	\$7,633	\$8,015	\$8,415	\$8,792	\$9,055	\$9,327	\$9,607	\$9,895	\$10,192
Present Value Factor	0.8981	0.7245	0.5844	0.4714	0.3803	0.3068	0.2475	0.1996	0.1610	0.1299
Present Value of DFNCF	6,232	5,530	4,684	3,967	3,343	2,778	2,308	1,918	1,593	1,324
Total Present Value of Discrete Period DFNCF					33,679					
Calculation of Terminal Value										
Normalized Terminal Year					10,447					
Capitalization Rate					21.5%					
Terminal Value of DFNCF					48,665					
Present Value Factor at 24.0%					0.1299					
Present Value of Terminal Value					6,321					
Valuation Summary										
Total Present Value of Discrete Period DFNCF					33,679					
Present Value of Terminal Value					6,321					
Fair Value of Invested Capital					\$40,000					
IRR					24.0%					

often end up consuming all of the cash flows used in calculating the fair value of customer relationships. In such cases, it is often necessary to explore other valuation methods for the customer relationships, so that the value of the customers reflects economic assumptions a market participant typically would have made regarding those assets. While it is very rare to have both economic obsolescence and residual goodwill, it is not uncommon to have both economic obsolescence and value within the identified intangible assets such as customer relationships, trademarks, or patented technology.

Rates of return

Commonly, as part of a purchase price allocation analysis, it would be expected that three rates of return would be in alignment: weighted average cost of capital (WACC, or a hypothetical return on the subject company calculated by the appraiser), implied rate of return (IRR, the rate of return determined based on the projections used to price the acquisition and the ultimate purchase price), and weighted average return on assets (WARA, the aggregate rate of return required for each acquired asset weighted in proportion to the fair value of that asset to the purchase price).

In a bargain purchase scenario, however, these rates would often initially not be in alignment without further efforts to reconcile them. This scenario typically observes the following relationship: $IRR > WACC > WARA$. The IRR tends to be the highest because the subject company was purchased at a low price (hence a “bargain purchase”), and this indicates that the buyer is requiring an above market rate of return on the investment. The WACC is in

the middle because it is calculated without regard to the purchase price and reflects return requirements of market participants; hence, it is unaffected by the nature of the acquisition. The WARA tends to be the lowest rate of return because in a bargain purchase, there is no goodwill, which typically has the highest required rate of return of the acquired assets due to its inherent riskiness, while goodwill would be present in the normal market assumptions associated with the WACC. This difference in the asset mix pushes toward a lower WACC, with all else being equal. In a bargain purchase scenario, the reconciliation of these differences can be a complex process.

Independent valuation

To further support the evidence of a bargain purchase, the appraiser may calculate his/her own fair value of the business enterprise using market participant data. If the appraiser determines that the future cash flows of the business are market participant cash flows (i.e., exclude buyer-specific synergies), he/she can determine the fair value of the subject company using a market participant WACC. Using the same two scenarios of an acquisition representing fair value with residual goodwill, and a bargain purchase scenario with negative goodwill, he/she can demonstrate that the purchase price is not representative of fair value.

One method of calculating an independent fair value of the subject company is through a discounted cash flow (DCF) analysis. Table 3 shows the forecasted debt-free net cash flow (DFNCF) of the subject company. Using the DFNCF, a long-term growth rate of 2.5%, and the

Table 4
Fair Value of Invested Capital using the Market Participant Cash Flows and the WACC

	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025
DFNCF	\$6,939	\$7,633	\$8,015	\$8,415	\$8,792	\$9,055	\$9,327	\$9,607	\$9,895	\$10,192
Present Value Factor	0.9262	0.7945	0.6815	0.5846	0.5014	0.4301	0.3690	0.3165	0.2715	0.2329
Present Value of DFNCF	6,427	6,064	5,462	4,920	4,409	3,895	3,441	3,041	2,686	2,374
Total Present Value of Discrete Period DFNCF					42,718					
Calculation of Terminal Value										
Normalized Terminal Year					10,447					
Capitalization Rate					14.1%					
Terminal Value of DFNCF					74,206					
Present Value Factor at 16.6%					0.2329					
Present Value of Terminal Value					17,282					
Valuation Summary										
Total Present Value of Discrete Period DFNCF					42,718					
Present Value of Terminal Value					17,282					
Fair Value of Invested Capital					\$60,000					
WACC					16.6%					
Long Term Growth Rate					2.5%					

purchase price of \$40 million, we see that the IRR is 24.0%. Let's assume that the appraiser calculated a WACC of 16.6% for the subject company. If we assume that these cash flows are market participant cash flows, Table 4 shows that using the same DFNCF, a long-term growth rate of 2.5%, and the WACC, we find that the fair value of the subject company is \$60 million. The IRR of 24.0% versus the WACC of 16.6% demonstrates that the acquirer of the subject company purchased the company at a bargain price, because the IRR is an above-market return when compared to the WACC.

In addition to a DCF analysis, the appraiser could calculate the fair value of the subject company through a market-based approach. By looking at the trading multiples of public comparable companies, or transaction

multiples of similar acquired companies, the appraiser could determine that the implied multiples of the transaction are below the range of multiples that are representative of fair value.

As previously mentioned, reconciling the rates of return in a bargain purchase scenario can be a complex process. Tables 5 and 6 show the WARA under each scenario. In Table 5, using the appraiser's independent valuation of \$60 million, the WARA is equal to the WACC, and the return on goodwill has the highest return of all of the acquired assets. When using the purchase price in Table 6, the WARA is much lower than the WACC, and the previously mentioned relationship holds. It is important to note that the selected rates of return in Tables 5 and 6 are illustrative only; the return on goodwill

Table 5
WARA with Positive Goodwill

Assets	Fair Value	% of Total	Rate of Return, %
Net Working Capital	\$15,000	25.0	5.0
Personal Property	10,000	16.7	6.0
Real Property	1,000	1.7	7.0
Identified Intangible Assets			
Patents and Technology	5,000	8.3	20.0
Trade Name	7,000	11.7	20.0
Customer Relationships	4,000	6.7	20.0
Unallocated Intangible Assets			
Trained and Assembled Workforce	500	0.8	17.0
Goodwill	17,500	29.2	30.0
Purchase Price	\$60,000	100.0	16.6

Table 6
WARA with Negative Goodwill

Assets	Fair Value	% of Total	Rate of Return, %
Net Working Capital	\$15,000	37.5	5.0
Personal Property	10,000	25.0	6.0
Real Property	1,000	2.5	7.0
Identified Intangible Assets			
Patents and Technology	5,000	12.5	20.0
Trade Name	7,000	17.5	20.0
Customer Relationships	4,000	10.0	20.0
Unallocated Intangible Assets			
Trained and Assembled Workforce	500	1.3	17.0
Fair Value of Identified Assets	\$42,500	106.3	11.8
Goodwill	(2,500)	-6.3	
Purchase Price	\$40,000	100.0	

is an implied return based on the rates of return of the other acquired assets and the WARA equaling the WACC.

Implications

The primary implication of a bargain purchase is the gain to the buyer if in fact the purchase actually was a “bargain” relative to the fair value of the acquired business. As discussed earlier, ASC 805 mandates that a bargain purchase gain be recognized at the time of acquisition and recorded as extraordinary income at the date of purchase. However, it is important to note that this is a gain for generally accepted accounting principles accounting only, and hence this gain would not be included in the calculation of taxable income.

How to Handle a Bargain Purchase

In dealing with a bargain purchase, much of the natural support of market expectations provided by the transaction itself is missing. Therefore, extra analysis is necessary to ensure that the purchase price allocation represents the views of market participants in an accurate and supportable manner. As bargain purchases are somewhat rare, auditors tend to be skeptical of such a conclusion. Here are a few pointers to help smooth the process.

Find out as soon as possible

In doing the initial scoping of the appraisal, the appraiser should ask the client and/or auditor about the transaction and if a bargain purchase is anticipated. This can help to align expectations and generate a more

thoughtful information request list as well as guide the discussions with company management and the auditors. This initial scoping may help to determine the rationale for the bargain purchase, which will provide support for the analysis and report.

Overcommunicate

If the analysis reveals a potential bargain purchase that was not anticipated, it is important to notify all relevant parties as soon as possible. This will help progress the appraisal when it is in audit review, as many of the questions auditors may have can be answered up front as opposed to during the review. If a fixed asset appraisal is being done as well, this should be communicated early on to the fixed asset appraisers, as they may need to apply economic obsolescence to the fixed assets.

Reconcile the rates of return

In looking at the three rates of return (IRR, WACC, and WARA), the appraiser should be able to explain why they are equal or far apart. In most cases, a separate valuation of the business should be performed to determine the actual fair value of the subject enterprise in order to support the rates of return used in the valuation of the individual assets.

Communicate the implications

Once it has been established that the transaction is a bargain purchase, it is important that the management of the subject company know how this will impact financial performance going forward.

STATUTORY FAIR VALUE IN DISSENTING SHAREHOLDER CASES: PART I

Gilbert E. Matthews, MBA, CFA

The predominant standard of value employed by state courts to determine the value of minority shares in appraisal cases is fair value, which is determined by state law. In most states, fair value is the shareholder's pro rata portion of the value of a company's equity. This measure of value differs from fair market value, third-party sale value, and fair value for GAAP purposes. This article discusses the valuation approaches accepted by the courts, focusing on the Delaware courts' views as to how fair value is assessed, and contrasts Delaware's views with those of other jurisdictions that differ from Delaware in their approach to fair value.

I. Fair Value as the Standard of Value in Dissenting Shareholder Cases

In this article I address the predominant standard of value, *fair value*, employed by state courts to determine the value of minority shares in appraisal (dissent) cases.¹ Appraisal statutes in most states expressly or effectively stipulate that the minority's shares are to be valued at "fair value." To understand fair value as a standard of measurement, it must be contrasted to the standards of value called *fair market value* and *third-party sale value*, as will be discussed in this article and continued in the next issue (Part II).² Please refer to the article outline on page 16.

Appraisal cases are governed by state law, that is, relevant corporate law statutes, the judicial interpretations of those statutes, and the courts' holdings under their general equitable authority, even when the state lacks corresponding statutes.³ Although fair value is now the state-mandated or accepted standard for judicial valuations for appraisal in almost all states, differing

Gilbert E. Matthews, MBA, CFA, is Chairman of Sutter Securities Incorporated in San Francisco, California. He headed the fairness opinion practice at Bear Stearns in New York for twenty-five years. He is on the editorial review board of *Business Valuation Resources*.

¹Since shareholders who *dissent* from a transaction are entitled to appraisal of their shares, the terms *dissenters' rights and appraisal rights* are interchangeable.

²*Fair value* for appraisal is distinct from fair value for US GAAP. As defined in FAS 157, fair value for accounting purpose is a form of *fair market value*.

³Douglas K. Moll, "Shareholder Oppression and 'Fair Value': Of Discounts, Dates, and Dastardly Deeds in the Close Corporation," 54 *Duke L.J.* 293 (2004): 310.

interpretations of its meaning and measurement have evolved through legislative changes and judicial interpretation.

The model statutes proposed by the American Bar Association (ABA) and the American Law Institute (ALI), together with Delaware corporate laws on appraisals, have greatly influenced a majority of state statutes. The ABA and the ALI have developed definitions of fair value that are set forth in the ABA's *Model Business Corporation Act* (MBCA)⁴ and the ALI's *Principles of Corporate Governance*.⁵ Although statutes and legal organizations have both contributed to the development of the fair value standard, the courts' decisions in dissenting shareholder cases are central to its definition.

Delaware's appraisal statute explicitly mandates fair value as the measure of value, and the Delaware Supreme Court clarified its meaning in *Tri-Continental*⁶ in 1950. *Fair value* was defined as the value that had been taken from the dissenting shareholder:

The basic concept of value under the appraisal statute is that the stockholder is entitled to be paid for that which has been taken from him, *viz.*, his proportionate interest in a going concern. By value of the stockholder's proportionate interest in the corporate enterprise is meant the true or intrinsic value of his stock which has been taken by the merger. In determining what figure represents this true or intrinsic

⁴The MBCA is a model code designed for use by state legislatures in revising and updating their corporation statutes. It was initially published by the ABA in 1950 and revised in 1971, 1984, and 1999. There were amendments with respect to appraisals in 1969, 1978, and 2008.

⁵The *Principles of Corporate Governance* were written to "clarify the duties and obligations of corporate directors and officers and to provide guidelines for discharging those responsibilities in an efficient manner, with minimum risks of personal liability." ALI, *Principles of Corporate Governance* (Philadelphia: American Law Institute Publishers, 1992) at President's Foreword, xxi.

⁶*Tri-Continental v. Batty*, 74 A.2d 71 (Del. 1950) ("*Tri-Continental*").

Outline

PART I

- I. Fair Value as the Standard of Value in Dissenting Shareholder Cases**
 - A. Appraisal Rights Today
 - B. Fair Value as Defined by Various Authorities and Statutes
 - C. The Importance of Delaware
 - D. Appraisal Arbitrage
- II. Fair Value in Delaware**
 - A. Delaware Fair Value Standards
 - B. Fair Value Is Proportionate Share of Equity Value
 - C. Fair Value Is Going-Concern Value
 - D. Fair Value May Be Greater or Less than Transaction Price
 - 1. Arm's-Length Price May Be Fair Value
 - 2. Arm's-Length Price May Be Greater than Fair Value
 - 3. Arm's-Length Price May Be Less than Fair Value
 - E. Fair Value Is Based on How the Company Is Being Managed Prior to Transaction
 - F. Fair Value Excludes Synergies Resulting from the Transaction but Includes Enhancements Obtainable by Current Controller
 - G. Fair Value Includes Changes Contemplated by Management
 - 1. Usurped Corporate Opportunities
 - 2. Improper Benefits to Control Shareholder
 - 3. Improvements Not Dependent on Merger
 - H. Fair Value Is Not Third-Party Sale Value
 - I. Taxes Are Considered Only If They Are "Operative Reality"
 - J. Tax-Affecting S Corporations
- III. Valuations in Other Jurisdictions May Differ from Delaware**
 - A. Appraisals under the MBCA Could Be Higher than in Delaware
 - B. Some States Consider Financial Control Value
 - C. Appraisals by the Controller of the Currency

PART II

- IV. Customary and Current Valuation Methods**
- V. Fair Value Normally Excludes Discounts and Premiums**
 - A. Levels of Value
 - a. Why Minority and Marketability Discounts and Control Premiums Are Widely Rejected
 - B. Valuation Adjustments at the Entity Level
 - C. Minority Discounts Are Impermissible
 - D. Marketability Discounts Are Impermissible in Most States
 - 1. The MBCA and the ALI
 - 2. Most States Now Reject Marketability Discounts
 - 3. New York Generally Accepts Marketability Discounts
 - E. Control Premiums at the Entity Level
- VI. States Using a Standard Other Than Fair Value**
- VII. Valuation Date in Appraisal Cases**
- VIII. Summary**

value, the appraiser and the courts must take into consideration all factors and elements which reasonably might enter into fixing the value.⁷

This concept of value has since been cited in numerous appraisal cases as the basic standard. In recent years, most jurisdictions have accepted the position that appraisal should measure *what has been taken from the shareholder* and that this amount is a *pro rata* share of the value of the company as a going concern.

The appraisal action is a "limited legislative remedy which is intended to provide minority shareholders who

dissent from a merger asserting the inadequacy of the [consideration], with an independent judicial determination of the fair value of their shares."⁸ Dissenting minority shareholders may petition for appraisal under a state statute, commonly known as *appraisal* or *dissenters'* rights. Shareholders customarily have appraisal rights when they are involuntarily cashed out in a merger or consolidation, but some states also permit dissenters to seek appraisal in other circumstances, such as a sale of assets, recapitalization, stock-for-stock merger, amendments to articles of incorporation, or other major changes

⁷*Id.* at 72.

⁸*Alabama By-Products Corp. v. Neal*, 588 A.2d 255, 256 (Del. 1991).

to the nature of their investment. In an appraisal action, the exclusive remedy is cash.

Importantly, defining *fair value* as a proportionate share of a company's value, as Delaware did in *Tri-Continental*, differentiates it from the other two relevant standards of value: *fair market value* and *third-party sale value*. Professors Hamermesh and Wachter write:

“[T]he measure of ‘fair value’ in share valuation proceedings is superior, in both fairness and efficiency, to its two main competitors, [fair] market value and third-party sale value.”⁹

Fair market value is “the price at which the property would change hands between a willing buyer and a willing seller when the former is not under any compulsion to buy and the latter is not under any compulsion to sell, both parties having reasonable knowledge of relevant facts.”¹⁰ In contrast, fair value is used in statutory appraisals where the seller is not a willing seller, is compelled to sell, and has less knowledge of the relevant facts than does the buyer.

When fair market value is used in (for example) tax cases, substantial discounts for the minority's lack of control and lack of marketability are often applied to the value of minority shares. Courts have noted that a fair market value valuation based on such discounts would be less than the value of the minority shareholders' proportionate interest in the company. With a fair market valuation, the controller (or majority) would reap a windfall at the expense of the minority. Consequently, statutes and judicial interpretations in most states now reject minority or marketability discounts in the determination of fair value.

On the other hand, if the courts used the standard of *third-party sale value*, those shares may be valued at a level that would usually be higher than fair value. An augmented value results when third-party sale price includes additional elements of value resulting from the transaction, such as synergies. Minority shareholders are not entitled to those incremental values. Hamermesh and Wachter explain:

Third-party sale value necessarily derives from transactions in which corporate control is acquired. The Delaware cases establish, however, that it is *the nature of the enterprise itself* at the time of the merger that is the key parameter in the valuation exercise [emphasis in original]. Because the prices paid in such transactions reflect elements of value created by the transaction—notably synergies—that would not other-

wise exist in the enterprise itself, the use of such prices in determining fair value conflicts with the statutory mandate that “any element of value arising from the accomplishment or expectation of the merger or consolidation” must be excluded.¹¹

These writers maintain that the fair value standard is fairer to opposing parties in a dispute than either fair market value or third-party sale value because fair value attempts to balance the dangers that lie in either direction: on one side, the danger of awarding a windfall to an opportunistic controller who has forced out the minority shareholders; on the other side, the danger of incentivizing litigation by minority shareholders attempting to capture value from controllers whose energies and abilities have resulted in increased company value through a synergistic transaction. They posit that fair value strikes the best balance for valuing what was taken from the minority by awarding them the pro rata share of the existing company's going-concern value, that is, the present value of the cash flows to be generated from the corporation's existing assets *plus its reinvestment opportunities*.¹²

To further understand the issues and complexity surrounding fair value in appraisal and fiduciary duty cases, this article examines what elements of value are addressed by the courts in their determination of fair value, and looks at how various courts address current valuation concepts and techniques.

A. Appraisal rights today

Currently, the ABA and the ALI recognize various events that can trigger dissenters' rights. States have adopted their own triggering events in their statutes, and these may have developed differently from those of the MBCA and the *Principles of Corporate Governance* because of each state's corporate law history. Some common triggers contained in the MBCA and the state statutes include:

- Merger
- Share exchange
- Disposition of assets
- Amendment to the articles of incorporation that creates fractional shares
- Any other amendment to the articles from which shareholders may dissent
- Change of state of incorporation
- Conversion to a flow-through, unincorporated or nonprofit entity

⁹Lawrence A. Hamermesh and Michael L. Wachter, “Rationalizing Appraisal Standards in Compulsory Buyouts,” 50 *Boston College L. Rev.* 1021 (2009): 1021 (“Rationalizing Appraisal Standards”). In this article, we extensively cite these professors' expert writings on appraisal and fair value.

¹⁰IRS Rev. Ruling 59-60, §2.02.

¹¹Hamermesh and Wachter, “Rationalizing Appraisal Standards” at 1028, quoting 8 DEL. CODE ANN. §262(h).

¹²*Id.* at 1022.

In practice, a majority of appraisal cases today arise when control shareholders squeeze out minority shareholders for cash.

Professor Robert Thompson points out that the appraisal remedy serves as a check against opportunism:

Now the remedy serves as a check against opportunism by a majority shareholder in mergers and other transactions in which the majority forces minority shareholders out of the business and requires them to accept cash for their shares. In earlier times, policing transactions in which those who controlled the corporation had a conflict of interest was left to the courts through the use of fiduciary duty or statutes that limited corporate powers. Today, that function is left for appraisal in many cases.¹³

Thompson makes the case that that several statutory appraisal provisions work counter to providing fairness in the opportunism context:

- Excluding from the fair value calculation any appreciation or depreciation attributable to the merger transaction;
- Requiring minority shareholders seeking appraisal to take four or more separate legal steps to perfect the remedy (and withdrawing relief if the actions are not perfect);
- Excluding appraisal when shares are traded on a public market;¹⁴ and
- Making appraisal an exclusive remedy even when the valuation remedy does not include loss from breaches of fiduciary duty.¹⁵

Dissenters are required to follow precisely the complex timing and other requirements of state law in a process referred to as perfecting dissenters' rights. The process and timetable of these events vary from state to state, but in most cases are strictly enforced. A company's board of directors is required to give notice (commonly in a proxy or information statement) of a contemplated corporate action from which shareholders may dissent. Dissenters must then decline the consideration and demand payment of their shares in a notice to the board prior to the action. This dissent triggers an appraisal. Upon notice of dissent, dissenters relinquish all rights except the right to receive payment of the fair value of their shares (plus interest from the valuation date) and, in many states, will receive no payment until the conclusion or settlement of litigation. (In some states, however, the company must [or may elect to] pay or put into escrow, the amount that it

contends to be fair value, as recommended by the MBCA.) Furthermore, dissenters become unsecured creditors of the company or its successor, which often is a highly leveraged entity. In an unpublished 1999 Alabama appraisal, *Delchamps, Inc. v. Kuykeldall*, in which the author was an expert witness, petitioners were awarded an amount materially above the transaction price, but the highly leveraged acquirer filed for bankruptcy shortly after the verdict without paying petitioners.

B. Fair Value as defined by various authorities and statutes

"Fair value" is the standard of value for statutory appraisal in forty-eight states and the District of Columbia.¹⁶ State statutes vary, but most draw inspiration from the MCBA (1969) and the later revised MBCAs (1984, 1999, and 2008). The definitions in the various iterations illustrate the evolution of the fair value standard.

The 1969 MBCA set out that "fair value" was to be the measure by which the minority shareholder was to be paid for his or her shares, but it provided no details on fair value's definition. It stated that:

[S]uch corporation shall pay to such shareholder, upon surrender of the certificate or certificates representing such shares, the fair value thereof as of the day prior to the date on which the vote was taken approving the proposed corporate action, excluding any appreciation or depreciation in anticipation of such corporate action.¹⁷

In 1984, the ABA issued a revised MBCA, adding important additional concepts to the definition of fair value. It excluded from the value of minority shares the synergy value of the objected-to transaction "unless exclusion would be inequitable." It reads: "The value of the shares immediately before the effectuation of the corporate action to which the dissenter objects, excluding any appreciation or depreciation in anticipation of the corporate action *unless exclusion would be inequitable* [newly added language in italics]."¹⁸

The 1984 definition provided a guideline, however nonspecific, by which fair value should be determined. The company should be valued without any of the effects of the transaction unless that exclusion would be unfair. The passage did not give instructions on what method or

¹³Robert B. Thompson, "Exit, Liquidity, and Majority Rule: Appraisal's Role in Corporate Law," 84 *Georgetown L. Rev.* 1 (Nov. 1995): 4.

¹⁴Shareholders of publicly traded companies are denied appraisal rights in all transactions in thirteen states, in transactions other than interested party transactions in eleven states, and in stock-for-stock transactions in Delaware and twelve other states. See Gilbert E. Matthews and Michelle Patterson, "Public Shareholders, Fair Value, and the 'Market-Out Exception' in Appraisal Statutes," 21 *Business Valuation Update* 17, Feb. 2015: 17–25.

¹⁵Thompson at 5.

¹⁶The exception are California, whose appraisal statute specifies *fair market value*, and Ohio, whose statute uses the phrase *fair cash value*. Wisconsin uses *fair value* for most transactions but uses *market value* for certain related party transactions. See "States Using a Standard Other than Fair Value" in Part II.

¹⁷1969 MBCA §13.01.

¹⁸1984 MBCA. This provision is currently in many state statutes. See, e.g., COLO. REV. STAT. §7-113-101(4).

valuation technique should be utilized to determine the fair value, nor does it define *inequitable*. The intentional ambiguity in this definition allowed for wide interpretation of the assumptions that underlie this standard of value. Comments published by the ABA explained that this definition left the matter to the courts to determine “the details by which fair value is to be determined within the broad outlines of the definition.”¹⁹

Although most state statutes use a definition of fair value from a version of the MBCA, some states have utilized the definition provided by the ALI. In its *Principles of Corporate Governance*, published in 1992, the ALI defined fair value as

the value of the eligible holder’s proportionate interest in the corporation, without any discount for minority status or, absent extraordinary circumstances, lack of marketability. Fair value should be determined using the customary valuation concepts and techniques generally employed in the relevant securities and financial markets for similar businesses in the context of the transaction giving rise to appraisal.²⁰

Following the development of substantial case law on valuation of dissenters’ shares, as well as the publication of the ALI’s *Principles of Corporate Governance*, the MBCA was revised in 1999 so that the definition of fair value reads:

The value of the shares immediately before the effectuation of the corporate action to which the shareholder objects using customary and current valuation concepts and techniques generally employed for similar businesses in the context of the transaction requiring appraisal, and without discounting for lack of marketability or minority status except, if appropriate, for amendments to the certificate of incorporation.²¹

This definition (unchanged in 2008) affords a more inclusive view of opportunities, which may affect the determination of fair value. It mirrors the ALI’s *Principles of Corporate Governance* in that it adds important concepts to the framework: the use of customary and current valuation techniques, and the explicit rejection of the use of marketability and minority discounts.²²

Other states, including Delaware, have developed their own definitions of fair value²³ or have used different standards of value in their statutes. It should be recognized

that “there has been a constructive symbiosis between the MBCA and Delaware.”²⁴ The amended MBCA’s language calling for “using customary and current valuation concepts and techniques generally employed” is substantially the standard that Delaware had adopted in *Weinberger*²⁵ in 1983. A review of published appraisal decisions indicates that, in practice, many state courts had already been following the *Weinberger* standard.

The diversity among states in their definitions of fair value combined with the complexity of each state’s statutes compels the valuation expert to closely consult counsel for guidance before undertaking either appraisal testimony or a fairness assessment.

C. The importance of Delaware

More than half of all publicly traded US corporations and about two-thirds of the Fortune 500 are incorporated in Delaware,²⁶ and a major portion of corporate litigation takes place in Delaware. Delaware has by far the most extensive body of case law. The Delaware General Corporation Law is comprehensive and widely understood. Delaware has a trial court, the Court of Chancery, which is dedicated to equity cases and is knowledgeable and widely respected.

D. Appraisal arbitration

Much of the rise in appraisal petitions is attributable to “appraisal arbitration,” in which investors, primarily hedge funds, purchase shares eligible for appraisal to obtain returns through an appraisal proceeding rather than through the merger itself.²⁷ This practice has been particularly effective in related party transactions.

In 2007 the Court of Chancery held in *Transkaryotic*²⁸ that appraisal rights are available to investors who hold shares in “street name” at the date of the stockholder meeting, even if they had purchased shares after the record date:

[S]tockholders ... [learn] key information that will help them evaluate the merger ... only when they see the company’s proxy statement, which also ... is generally

¹⁹ABA, *A Report of the Committee of Corporate Laws*, “Changes in the Revised Model Business Corporation Act: Amendments Pertaining to Close Corporations,” 54 *Bus. Lawyer* 209 (Nov. 1998).

²⁰*Principles of Corporate Governance* at §7.22.

²¹1999 MBCA §13.01.

²²The exception permitting discounts for certain amendments to the certificate of incorporation has minimal impact.

²³Some states have explicitly accepted the Delaware definition in their case law. *See, e.g., Vortex Corp. v. Denkewicz*, 334 P.3d 734 (Ariz. App. 2014).

²⁴Jeffrey M. Gorris, Lawrence A. Hamermesh, and Leo E. Strine, Jr., “Delaware Corporate Law and the Model Business Corporation Act: A Study in Symbiosis,” 74 *Law & Contemp. Prob.* 107 (2011): 107.

²⁵*Weinberger v. UOP, Inc.*, 457 A.2d 701 (Del. 1983) (“*Weinberger*”).

²⁶John L. Reed and Ashley R. Altschuler, “Delaware Corporate Law and Litigation: What Happened in 2014 and What It Means for You in 2015,” available at <https://www.dlapiper.com/en/us/insights/publications/2015/01/delaware-corporate-litigation-review-2014-2015/>.

²⁷Charles R. Korsmo and Minor Myers, “Appraisal Arbitrage and the Future of Public Company M&A,” 92 *Wash. U. L. Rev.* 1551 (2015).

²⁸*In re Appraisal of Transkaryotic Therapies, Inc.*, 2007 Del. Ch. LEXIS 57 (May 2, 2007).

when they learn of the record date—and that record date is almost always publicly disclosed after it has passed.²⁹

As a result of *Transkaryotic*, hedge funds and others can evaluate potential appraisal claims and choose to accumulate shares between the record date and the closing date.³⁰ The decision widened the door for appraisal arbitrage. Numerous hedge funds have actively pursued this activity, and some hedge funds have been created specifically for this purpose.

Another factor is the interest rate applicable to Delaware appraisals. Shareholders are awarded interest at a rate of 5% over the Federal Reserve discount rate, compounded quarterly, on the fair value of their shares. This generous interest rate has contributed to the rise of appraisal arbitrage in the low-interest-rate environment that has prevailed since 2008. As of August 1, 2016, Delaware amended its appraisal statute to permit companies to reduce interest accruals by prepaying to petitioners an amount chosen by the company.

In the past decade, appraisal arbitrage has resulted not only in an increase in the portion of eligible Delaware transactions where appraisal is sought, but also in a material increase in the aggregate number of shares involved in appraisals of public companies. The increase is attributable not only to hedge fund activity but also to an increased proclivity of mutual funds to seek appraisal in related party transactions.

II. Fair Value in Delaware

A. Delaware fair value standards

The concept of fair value under Delaware law is not equivalent to the economic concept of fair market value. Rather, the concept of fair value for purposes of Delaware's appraisal statute is a largely judge-made creation, freighted with policy considerations.³¹

The Delaware Supreme Court developed the standards for valuations in appraisal cases in four seminal cases: *Tri-Continental* (1950), *Sterling v. Mayflower* (1952),³² *Weinberger* (1983), and *Cavalier* (1989):³³

- *Tri-Continental* described fair value as that which has been taken from the shareholder and stated that

fair value should be determined based on facts known or knowable at the valuation date.³⁴

- *Sterling v. Mayflower* stated that the proper test of fairness was whether the “minority stockholder will receive *the substantial equivalent in value of the shares he held* [emphasis added].”³⁵
- *Weinberger* permitted the use of valuation techniques customarily accepted in the financial community and endorsed forward-looking valuation approaches.³⁶
- *Cavalier* confirmed that discounts for lack of marketability or minority interest should not be applied in calculating fair value.³⁷

Subsequent case law is based on these principles. Delaware's appraisal statute³⁸ has been further clarified in numerous decisions interpreting how fair value is to be determined and explaining which factors should be considered and excluded. We discuss these developments in this section.

B. Fair value is proportionate share of equity value

The Delaware appraisal statute states that “the Court shall determine the fair value of the shares exclusive of any element of value arising from the accomplishment or expectation of the merger or consolidation. . . . In determining such fair value, the Court shall take into account all relevant factors.”³⁹ Interpreting the statute, the Delaware Court of Chancery ruled in 1988 that a dissenting shareholder was entitled to a pro rata share of the equity value of the company:

Under §262 [of the Delaware General Corporation Law], the dissenting shareholder is entitled to his proportionate interest in the overall fair value of the corporation, appraised as a going concern. The amount of the holdings of a particular dissenting stockholder is not relevant, except insofar as they represent that shareholder's proportionate interest in the corporation's overall “fair value.” That a particular dissenting stockholder's ownership represents only a minority stock interest in a corporation is, therefore, legally immaterial in determining the corporation's “fair value.”⁴⁰

The Delaware Supreme Court upheld the lower court decision, explaining that if minority shareholders did not

²⁹Korsmo and Myers, “Reforming Modern Appraisal Litigation,” 41 *Del. J. Corp. L.* (2017) [forthcoming], available at <http://ssrn.com/abstract=2712088>, p. 70.

³⁰Shareholders as of the record date are eligible for appraisal even if they bought their shares after the announcement date.

³¹*Finkelstein v. Liberty Media, Inc.*, 2005 Del. Ch. LEXIS 53 (Apr. 25, 2005) at *39.

³²*Sterling v. Mayflower Hotel Corp.*, 93 A.2d 107 (Del. 1952).

³³*Cavalier Oil Corp. v. Harnett*, 564 A.2d 1137 (Del. 1989) (“*Cavalier*”). See discussion of *Cavalier* in “Fair value is proportionate share of equity value” below.

³⁴*Tri-Continental* at 72.

³⁵*Sterling v. Mayflower* at 110.

³⁶*Weinberger* at 713.

³⁷*Cavalier* at 1145.

³⁸DEL. CODE ANN. §262.

³⁹DEL. CODE ANN. §262(h).

⁴⁰*Cavalier Oil Corp. v. Harnett*, 1988 Del. Ch. LEXIS 28 (Feb. 22, 1988) at *27; *aff'd*, 564 A.2d 1137 (Del. 1989).

receive “the full proportionate value” of their shares, the majority shareholders would “reap a windfall”:

[T]o fail to accord to a minority shareholder the full proportionate value of his shares imposes a penalty for lack of control, and unfairly enriches the majority shareholders who may reap a windfall from the appraisal process by chasing out a dissenting shareholder, a clearly undesirable result.⁴¹

This decision clearly supports the pro rata share concept, ruling that in an appraisal the minority shareholder should not receive a lesser price for his shares because he or she does not share in the exercise of control of the corporation. Because minority shareholders are entitled to proportionate value, the controller cannot benefit disproportionately from forcing out the minority at a diminished price.

C. Fair value is going-concern value

The Delaware courts have consistently held that the best measure of fair value is *going-concern value*.⁴² In Delaware, the concept of going-concern value is based on earnings from existing assets *plus the value of anticipated reinvestment opportunities*: “[G]oing concern value must include not only the discounted free cash flow to be generated by the corporation’s current assets, but also the discounted free cash flow to be generated by the reinvestment opportunities anticipated by the corporation.”⁴³

A Delaware company is appraised *as it exists at the transaction date*, inclusive of its anticipated reinvestments. This concept of going-concern value is often referred to as *operative reality*. In the 2012 *Just Care* decision, the court cited precedent and wrote:

In an appraisal proceeding, “the corporation must be valued as a going concern based upon the ‘operative reality’ of the company as of the time of the merger.” The Court should consider “all factors known or knowable as of the Merger Date that relate to the future prospects of the Companies,” but should avoid including speculative costs or revenues.⁴⁴

D. Fair value may be greater or less than the transaction price

1. Arm’s-length price may be fair value

When considering fair value, the court may elect to give substantial weight to a price negotiated in an arm’s-length transaction. The Delaware Supreme Court wrote in 1999

⁴¹*Cavalier* at 1145.

⁴²Hamermesh and Wachter, “Rationalizing Appraisal Standards” at 1022.

⁴³*Id.*

⁴⁴*Gearreald v. Just Care, Inc.*, 2012 Del. Ch. LEXIS 91 (Apr. 30, 2012) (“*Just Care*”) at *21, citing *M.G. Bancorp., Inc. v. LeBeau*, 737 A.2d 513, 525 (Del. 1999) and *In re U. S. Cellular Operating Co.*, 2005 Del. Ch. LEXIS 1 (Jan. 6, 2005) at *56.

that “a merger price resulting from arm’s-length negotiations where there are no claims of collusion is a very strong indication of fair value.”⁴⁵ The court wrote in 2004:

In view of the market’s opportunity to price UFG directly as an entity, the use of alternative valuation techniques like a DCF [discounted cash flow] analysis is necessarily a second-best method to derive value. A DCF analysis depends heavily on an assumption about the cost of capital that rational investors would use in investing in UFG, and assumptions about the accuracy of UFG’s cash-flow projections. The benefit of the active market for UFG as an entity that the sales process generated is that several buyers with a profit motive, were able to assess these factors for themselves and to use those assessments to make bids with actual money behind them.⁴⁶

In 2010 it ruled that the Court of Chancery can decide whether or not to give any weight to the transaction price:

Requiring the Court of Chancery to defer—conclusively or presumptively—to the merger price, even in the face of a pristine, unchallenged transactional process, would contravene the unambiguous language of the statute and the reasoned holdings of our precedent. . . . [I]nflexible rules governing appraisal provide little additional benefit in determining “fair value.” . . . Appraisal is, by design, a flexible process.⁴⁷

In a 2013 decision involving an acquisition by a financial buyer, Vice Chancellor Sam Glasscock III rejected the experts’ analyses and appraised the dissenters’ shares at the transaction price less “the synergy value of the transaction, if any.”⁴⁸

Four 2015 Delaware decisions appraised companies at or minimally below arm’s-length transaction prices. Vice Chancellor Glasscock appraised a company at the transaction price, writing:

I note that my DCF value . . . is still below that paid by the actual acquiror without apparent synergies; it would be hubristic indeed to advance my estimate of value over that of an entity for which investment represents a real—not merely an academic—risk.⁴⁹

Vice Chancellor John Noble stated that “the Merger price appears to be the best estimate of value” and

⁴⁵*M.P.M. Enterprises, Inc. v. Gilbert*, 731 A.2d 790, 797 (Del. 1999). See also *Miller Bros. Industries, Inc. v. Lazy River Inv. Co.*, 272 A.D. 2d (N.Y. App. 2000); *Dermody v. Sticco*, 465 A.2d 948, 951 (N.J. Super. 1983).

⁴⁶*Union Ill. v. Union Financial*, 847 A.2d 340, 359, citing Barry M. Wertheimer, “The Shareholders’ Appraisal Remedy and How Courts Determine Fair Value,” 47 *Duke Law Journal* 613 (1998): 655 (“The best evidence of value, if available, is third-party sales value. If such evidence is not available, there is no choice but to resort to less precise valuation techniques.”).

⁴⁷*Golden Telecom, Inc. v. Global GT LP*, 11 A.3d 214, 218 (Del. 2010).

⁴⁸*Huff Investment Fund v. CKx, Inc.*, 2013 Del. Ch. LEXIS 262 (Nov. 1, 2013) at *48-*49; *aff’d*, 2015 Del. LEXIS 77 (Del., Feb. 12, 2015).

⁴⁹*In Re Appraisal of Ancestry.com, Inc.*, 2015 Del. Ch. LEXIS 21 (Jan. 30, 2015) at *76.

determined that the company's appraised value was the price negotiated in a transaction that was negotiated after an extensive marketing process.⁵⁰

Vice Chancellor Donald Parsons, Jr. rejected DCF because "the management projections that provide the key inputs to the petitioner's DCF analysis [were] not reliable;" he valued the company at 99% of the price paid in a hostile transaction⁵¹ and commented that "hypothetical statements about how much money someone allegedly would have paid, if they actually had the money to do so, . . . are significantly less probative" than "bids with actual money behind them."⁵²

Later in 2015, Vice Chancellor Glasscock valued a company acquired in an arm's-length transaction at the acquisition price even though it was 4% lower than his calculated DCF value:

I undertook my own DCF analysis that resulted in a valuation of BMC at \$48.00 per share. . . .

Taking these uncertainties in the DCF analysis—in light of the wildly-divergent DCF valuation of the experts—together with my review of the record as it pertains to the sales process that generated the Merger, I find the Merger price of \$46.25 per share to be the best indicator of fair value of BMC as of the Merger date.⁵³

In December 2016, Vice Chancellor Laster wrote:

If the merger giving rise to appraisal rights "resulted from an arm's-length process between two independent parties, and if no structural impediments existed that might materially distort the 'crucible of objective market reality,'" then "a reviewing court should give substantial evidentiary weight to the merger price as an indicator of fair value."⁵⁴

He concluded:

Small changes in the assumptions that drive the DCF analysis, however, generate a range of prices that starts below the merger price and extends far above it. My best effort to resolve the differences between the experts resulted in a DCF valuation that is within 3% of the Final Merger Consideration.

As noted, a DCF analysis depends heavily on assumptions. Under the circumstances, as in *AutoInfo* and *BMC*, I give 100% weight to the transaction price.⁵⁵

⁵⁰*Merlin Partners LP v. AutoInfo, Inc.*, 2015 Del. Ch. LEXIS 128 (Apr. 30, 2015) at *48.

⁵¹*Longpath Capital, LLC v. Ramtron Intl. Corp.*, 2015 Del. Ch. LEXIS 177 (June 30, 2015) at *2.

⁵²*Id.* at *87.

⁵³*Merion Capital LP v. BMC Software, Inc.*, 2015 Del. Ch. LEXIS 265 (Oct. 21, 2015) at *64–*65.

⁵⁴*Merion Capital LP v. Lender Processing*, 2016 Del. Ch. LEXIS 189 at *40–*41, quoting *Highfields Capital, Inc. v. AXA Financial, Inc.*, 939 A.2d 34, 42 (Del. Ch. 2007).

⁵⁵*Id.* at *89.

Laster did not adjust valuation for synergies because the respondent's expert "declined to offer any opinion on the quantum of synergies or to propose an adjustment to the merger price."⁵⁶

Vice Chancellor Joseph Slight's rejected the projections underlying the petitioners' DCF analysis and appraised the company at the deal price. He concluded:

In the wake of a robust pre-signing auction among informed, motivated bidders, and in the absence of any evidence that market conditions impeded the auction, I can find no basis to accept the Petitioners' flawed, post-hoc valuation and ignore the deal price. Nor can I find a path in the evidence to reach a fair value somewhere between the values proffered by the parties. And so I "defer" to deal price, not to restore balance after some perceived disruption in the doctrinal Force, but because that is what the evidence presented in this case requires.⁵⁷

A major law firm recently commented:

These cases suggest that the court is likely to apply a "merger price minus synergies" valuation if the sales process is thorough, effective and free from conflicts of interest. Additionally, the court has been more willing to defer to the merger price if the other evidence, such as the petitioners' expert valuation evidence, is seen as problematic. For example, the court has viewed discounted cash flow analyses as less persuasive than the merger price when the reliability of the projections, discount rates and other inputs to the financial analysis are effectively called into question.⁵⁸

If the court determines that a purported third-party transaction was not arm's-length due to conflict of interest and/or improper actions by the buyer, the merger price is not credible evidence of fair value.⁵⁹

2. Arm's-length price may be greater than fair value

Fair value is going-concern value, while third-party sale value is the price that results from arm's-length negotiations. Because of synergies, as well as a buyer's ability to make changes to the operations and financial structure of a company, fair value is often less than third-party sale value:

Since the [appraisal] remedy provides going concern value and the shareholders [in an arm's-length transaction] are in fact receiving the higher amount, third-party sale value, the likely award in appraisal will be a lower amount than the

⁵⁶*Id.* at *90.

⁵⁷*In Re Appraisal of PetSmart, Inc.*, 2017 Del. Ch. LEXIS 89 (May 26, 2017) at *88.

⁵⁸Ronald N. Brown, III, and Keenan D. Lynch, "Recent Opinions Highlight Different Appraisal Valuation Methods Employed in Merger Transactions by Delaware Courts," in *Insights: The Delaware Edition*, Skadden, Arps, Slate, Meagher & Flom LLP, Nov. 17, 2016, p. 1.

⁵⁹*See, e.g., Just Care* at *15, n. 26; *Laidler v. Hesco Bastion Environmental, Inc.*, 2014 Del. Ch. LEXIS 75 (May 12, 2014) at *22.

dissenting shareholder will receive by voting in favor of the merger and taking the merger price.⁶⁰

Dissenting shareholders seek appraisal when they believe that the fair value of their shares is greater than the consideration that they were offered in the transaction. Dissenters have sometimes been awarded far more than the price they were originally offered, but this rarely happens when the buyer was a third party. The Supreme Court wrote in 1999:

A fair merger price in the context of a breach of fiduciary duty claim will not always be a fair value in the context of determining going concern value.

A merger price resulting from arms-length negotiations where there are no claims of collusion is a very strong indication of fair value. But in an appraisal action, that merger price must be accompanied by evidence tending to show that it represents the going concern value of the company rather than just the value of the company to one specific buyer.⁶¹

Importantly, dissenting shareholders have been awarded amounts lower than an arm's-length transaction price when the court determined that the transaction price included synergies and/or a control premium that should not have been included in fair value under Delaware law. A 2005 case concluded that the fair value of a company was \$2.74 per share, even though the minority shares had been acquired for \$3.31 in stock.⁶² A 2003 decision awarded the petitioner "the value of the Merger Price net of synergies,"⁶³ which gave the dissenters only 86% of the merger price. In 2012 a company acquired by a competitor was also appraised at 86% of the purchase price.⁶⁴ In 2017, a company was appraised using DCF at 82% of the value of the consideration at the time of the acquisition agreement (92% of the value of the consideration at closing).⁶⁵

3. Arm's-length price may be less than fair value

Although the Delaware courts have often equated fair value in appraisal with fair value in "entire fairness" cases,⁶⁶ there are several decisions in which the Court of Chancery stated appraisal valuations could be higher than

transaction prices that were deemed to meet the entire fairness standard.

In the long-running *Technicolor* case, in which there were more than twenty decisions over a twenty-one-year period, the Court concluded that "the \$23 per share received constituted the highest value reasonably available to the Technicolor shareholders"⁶⁷ and that the "transaction was in all respects fair to the shareholders of Technicolor."⁶⁸ In the appraisal case, however, the Supreme Court determined that the fair value of Technicolor was \$28.41.⁶⁹ This difference resulted in part because the valuation date for the appraisal was several months after the date that the transaction had been approved by Technicolor's board and because (as discussed in the next section) new higher projections were deemed relevant.

The Court of Chancery explained in 2014:

The entire fairness test is a standard of review, and the fair process aspect of the unitary entire fairness test is flexible enough to accommodate the reality that "[t]he value of a corporation is not a point on a line, but a range of reasonable values."⁷⁰ A price may fall within the range of fairness for purposes of the entire fairness test even though the point calculation demanded by the appraisal statute yields an award in excess of the merger price.⁷¹

A clear-cut example of the value for entire fairness being lower than appraised value is the 2016 *Dell* decision.⁷² In this prominent case, Dell, Inc. was taken private in a leveraged buyout (LBO) led by the founder, Michael Dell. The transaction price was \$13.88 per share, and Vice Chancellor J. Travis Laster awarded \$17.62 to the eligible dissenting shareholders based on his DCF analysis.⁷³ He wrote:

In this case, the Company's process easily would sail through if reviewed under enhanced scrutiny. The Committee and its advisors did many praiseworthy things. . . . In a

⁶⁰Hamermesh and Wachter, "The Fair Value of Cornfields in Delaware Appraisal Law," 31 *J. Corp. Law* 119 (2005): 142 ("Cornfields").

⁶¹*M.P.M. Enterprises, Inc. v. Gilbert*, 731 A.2d 790, 797 (Del. 1999).

⁶²*Finkelstein v. Liberty Media*, 2005 Del. Ch. LEXIS 53 at *84.

⁶³*Union Ill. 1995 Investment LP v. Union Financial Group, Ltd.*, 847 A.2d 340, 364 (Del. Ch. 2003).

⁶⁴*Just Care* at *1.

⁶⁵In *Re Appraisal of SWS Group, Inc.*, 2017 Del. Ch. LEXIS 90 (May 30, 2017) at *3-*4, *23. The value of the consideration was lower at closing because of a decline in the market price of the acquiror's stock.

⁶⁶Hamermesh and Wachter, "Rationalizing Appraisal Standards" at 1030.

⁶⁷*Cinerama, Inc. v. Technicolor, Inc.*, 663 A.2d 1134, 1142 (Del. Ch. 1994), *aff'd in relevant part*, *Cede, Inc. v. Technicolor, Inc.*, 663 A.2d 1156 (Del. 1995).

⁶⁸*Id.* at 1154.

⁶⁹*Cede, Inc. v. Technicolor, Inc.*, 884 A.2d 26, 30 (Del. 2005).

⁷⁰Quoting *Cede & Co. v. Technicolor*, 2003 Del. Ch. LEXIS 146 (Dec. 31, 2003) ("Technicolor 2003") at *2; *aff'd in part, rev'd in part on other grounds*, 884 A.2d 26.

⁷¹In *re Orchard Enterprises, Inc. Sh'holder Litig.*, 88 A.3d 1, 30 (Del. Ch. 2014). See also *Reis v. Hazelett Strip-Casting Corp.*, 28 A. 3d 442, 466 (Del. Ch. 2011) ("Reis"); *In re Trados Inc. Sh'holder Litig.*, 73 A.3d 17, 78 (Del. Ch. 2013); *Ancestry.com*, 2015 Del. Ch. LEXIS 21 at *50 ("[A] conclusion that a sale was conducted by directors who complied with their duties of loyalty is not dispositive of the question of whether that sale generated fair value."); *Merion Capital v. Lender Processing*, 2016 Del. Ch. LEXIS 189 at *43 ("Because the two inquiries are different, a sale process might pass muster for purposes of a breach of fiduciary claim and yet still constitute a sub-optimal process of an appraisal.")

⁷²In *re Appraisal of Dell Inc.*, 2016 Del. Ch. LEXIS 81 (May 31, 2016).

⁷³*Id.* at *167.

liability proceeding, this court could not hold that the directors breached their fiduciary duties or that there could be any basis for liability. But that is not the same as proving that the deal price provides the best evidence of the Company's fair value.⁷⁴

He observed the fairness opinions on which the directors relied were based on the values determined by the LBO models, and the valuations were constrained by the 20% minimum target internal rate of return used by LBO buyers.⁷⁵

E. Fair value is based on how the company is being managed prior to transaction

An important part of operative reality is how the company is being managed at the time of the transaction. A Delaware company being appraised is valued “as is” under its current management, not as it might be run by a different party:

The company, with all of its warts and diamonds, is valued in terms of the discounted free cash flow generated by the company's assets and reinvestment opportunities. In measuring the value of the warts and diamonds, the warts are valued as warts and the diamonds as diamonds. . . . The minority's claim is equal to the value of the shares into the future, and that value is a mix of the existing warts and diamonds.⁷⁶

It is important to note, however, that some “warts” can be disregarded; see “*Fair value includes changes contemplated by management*” below.

Management's plans, not those of an independent acquiror, are a company's operative reality. When a third-party buyer projected a higher growth rate for the target than did the target's management, the court determined that the appropriate input for the court's DCF calculation in an appraisal was the growth rate expected by the target's CEO, not the buyer's expectation.⁷⁷ Similarly, a company should be valued on its existing capital structure rather than on an optimal capital structure or the buyer's plans.⁷⁸

Actions planned by existing management prior to a squeeze-out merger are part of operative reality. Delaware normally excludes actions planned by a *third-party acquiror* before it acquires control. However, if control actually changes hands before a second-stage merger (a merger that squeezes out any minority shareholders whose shares were not acquired in an initial tender or exchange offer), the new control party's plans may be taken into account. The Delaware Supreme Court ruled in

its 1996 *Technicolor* decision that dissenting shareholders were entitled to benefit from changes being made or planned by a new management that had assumed control prior to the valuation date.⁷⁹ In that case, nontendering shareholders were squeezed out in the second-stage merger at the same price that had been paid for the bulk of the shares in November 1982 pursuant to a friendly tender offer by MacAndrews & Forbes. By the time the squeeze-out merger was consummated in January 1983, MacAndrews & Forbes had taken operating control. The Supreme Court ruled that MacAndrews & Forbes' plan, which involved disposing of certain unprofitable operations and increasing profit margins, was the operative reality and that the projections based on the buyer's plan should be the basis for the valuation.

The Court of Chancery's 2006 *Delaware Open MRI* decision stated, “The expansion plans for [additional MRI Centers] were clearly part of the operative reality of *Delaware Radiology* as of the merger date and under *Technicolor* and its progeny must be valued in the appraisal.”⁸⁰ It ruled:

The decision of [the control group] to cash out the [dissenters] at a price that did not afford it any of the value of the gains expected from [the additional MRI Centers] clearly bears on the fairness of the merger. Not only that, if the concept of opening [them] was part of the business plans of Delaware Radiology as of the merger date, then the value of those expansion plans must be taken into account in valuing Delaware Radiology as a going concern.⁸¹

The court added that “when a business has opened a couple of facilities and has plans to replicate those facilities as of the merger date, the value of its expansion plans must be considered in . . . determining fair value.”⁸²

The court may, however, reject the projected benefits of a planned expansion if it deems the project to be too speculative. In *Just Care*, a case appraising a company that operated a prison health-care facility in South Carolina, the financial projections included renovating a Georgia prison as a medical detention facility, the Court of Chancery distinguished this expansion from *Delaware Open MRI* and rejected the portion of the projection that related to a potential Georgia facility:

⁷⁹*Cede & Co. v. Technicolor*, 684 A.2d 289, 298-299 (Del. 1996) (“*Technicolor* 1996”). The MBCA now concurs with Delaware that changes prior to the squeeze-out merger in a two-step transaction should be included in fair value: “[I]n a two-step transaction culminating in a merger, the corporation is valued immediately before the second step merger, taking into account any interim changes in value.” (Official Comments to MBCA, §13.01 (2008), citing *Technicolor* 1996.)

⁸⁰*Delaware Open MRI Radiology Associates v. Kessler*, 898 A.2d 290, 316 (Del. Ch. 2006) (“*Delaware Open MRI*”).

⁸¹*Id.* at 313.

⁸²*Id.* at 314-315.

⁷⁴*Id.* at *88-89.

⁷⁵*Id.* at *90-91.

⁷⁶Hamermesh and Wachter, “Cornfields” at 143-144.

⁷⁷ *Crescent/Mach I Partnership, L.P. v. Dr Pepper Bottling Co. of Texas*, 2007 Del. Ch. LEXIS 63 (May 2, 2007) at *16-17 and *38.

⁷⁸*In re Radiology Associates, Inc. Litig.*, 611 A.2d 485, 493 (Del. Ch. 1991).

I find that the Georgia Case was too speculative to be included in the valuation of the Company as of the merger date. . . . [E]ven if the new facility was successful, there was a risk that Georgia would move its prisoners currently housed at the Columbia Center back to Georgia, thereby reducing the value of the Columbia Center.⁸³

Just Care could not undertake the expansion unilaterally without a decision by Georgia to move forward. The fact that the company was focused on expanding into Georgia and had taken actions in furtherance of that goal is insufficient to make the Georgia Case part of Just Care’s operative reality.⁸⁴

The court did consider Just Care’s planned expansion of its existing South Carolina facility but probability-weighted the calculated DCF value because of the uncertainty as to whether the state’s Department of Correction would proceed with the project. To risk-adjust the planned expansion, the court deducted 33.3% from the calculated value.⁸⁵

F. Fair value excludes synergies resulting from the transaction but includes enhancements obtainable by current controller

In Delaware, the fair value standard does not permit the benefits of synergies resulting from a transaction to be included in a going-concern valuation:

[S]ynergies dependent on the consummation of an arm’s-length acquisition or combination may not contribute to “fair value” in appraisal proceedings. Similarly, we conclude that operating efficiencies that arise from the acquiror’s new business plans are not properly included in determining “fair value,” as long as they are not operationally implemented before the merger, even though they derive solely from the enterprise’s own assets.⁸⁶

The Delaware Supreme Court has stressed that the value of synergies imbedded in a third-party purchase price should be excluded: “In performing its valuation, the Court of Chancery is free to consider the price actually derived from the sale of the company being valued, *but only after the synergistic elements of value are excluded from that price*” [emphasis added].⁸⁷

Under the going-concern “operative reality” concept, the court did not include the benefits to a near-bankrupt airline of the transaction’s cancellation of preferred stock, a debt restructuring, and a planned capital infusion.⁸⁸ The court declined to credit the existing shareholders for

benefits that could not have been achieved without the transaction. The acquiror’s future plans and projections assumed the completion of the merger, which was conditioned on concessions from creditors and the infusion of new capital. The court followed the practice of excluding from fair value any gains that would not have occurred but for the transaction. The court noted:

[T]he Concessions were not being implemented—and thus were not an “operative reality”—as of the merger date. On that date the only “operative reality” was that the parties had entered into a contract which provided that the Concessions would become operative if and when the merger closed.⁸⁹

Another example of a situation where the dissenter could not benefit from the consequences of the transaction was when a shareholder who refused to consent to the conversion of a C corporation into an S corporation was squeezed out.⁹⁰ The prospective tax benefits from the conversion were excluded from fair value because they could not have been achieved without the transaction: “Heng Sang’s conversion to an S corporation cannot be considered for valuation purposes, because without Ng’s consent it was not possible for Heng Sang to convert to subchapter S status before the merger, and Ng never granted his consent.”⁹¹

In contrast, if the controller can achieve the benefits without the transaction, the court may include the present value of those enhancements in fair value. In the 2004 *Emerging Communications* decision, the court concluded that the substantial posttransaction benefits that defendants attributed to the merger were in fact contemplated and achievable before the transaction. It ruled that the control shareholder could have achieved the benefits without the merger by other means, such as entering into a contract between his wholly owned private company and the public company he controlled:

The cost savings attributed to the consolidation were properly includable in the June projections, because they were contemplated well before the going private merger and could have been achieved without it. Prosser had identified potential consolidation savings before the Privatization occurred. Because Prosser controlled both ECM and ICC, *he had the power to accomplish those savings without a business combination, such as by intercompany contractual arrangements* [emphasis added]. To put it differently, the value achieved by Prosser’s existing pre-merger ability to effect those cost savings was an asset of ECM at the time of the Privatization merger.⁹²

⁸³Just Care at *21–*22.

⁸⁴Id. at *24.

⁸⁵Id. at *30.

⁸⁶Id. at 151.

⁸⁷Montgomery Cellular Holding Co. v. Dobler, 880 A.2d 206, 220 (Del. 2005).

⁸⁸Allenson v. Midway Airlines Corp., 789 A.2d 572, 585-6 (Del. Ch. 2001).

⁸⁹Id. at 583.

⁹⁰An election to become an S corporation requires the unanimous approval of its shareholders (INT. REV. CODE §1362(a)(2)).

⁹¹Ng v. Heng Sang Realty Corp., 2004 Del. Ch. LEXIS 69 (Apr. 22, 2004) at *18. The benefits of converting a C corporation to an S corporation were also excluded in *In re Sunbelt Beverage Corp. Sh’holder Litig.*, 2010 Del. Ch. LEXIS 1 (Jan. 5, 2010) at *53.

⁹²*In re Emerging Communications, Inc. Sh’holders Litig.*, 2004 Del. Ch. LEXIS 70 (Del. Ch. May 3, 2004) at *48–*49.

The court decided that the fact that the controller's ability to accomplish the cost savings before the merger was an asset of the public company at the merger date and that all shareholders were entitled to share pro rata in that benefit.

G. Fair value includes changes contemplated by management

If management is contemplating changes in the company at the time the relevant transaction is completed, or if new management has begun implementing its plans prior to a squeeze-out merger, the Court of Chancery will deem these changes to be operative reality. Professors Hamermesh and Wachter explain certain adjustments that Delaware will recognize in an appraisal:

[I]n appropriate circumstances in which a controlling shareholder is acquiring the minority shares, the courts have interpreted "fair value" to include elements of value that arise from assets or plans that were not in place operationally at the time of the merger. Those three areas . . . involve:

- (1) pro forma inclusion of assets not formally owned by the corporation at the time of the merger, but constructively attributed to the corporation because they had represented a corporate opportunity wrongfully usurped prior to the merger;
- (2) projections of post-merger returns in which actual costs are disregarded and excluded because they represent improper benefits to the controlling shareholder; and
- (3) operating improvements that the controlling shareholder implements following the merger but that do not depend causally upon the consummation of the merger.⁹³

The next three subsections discuss the three categories listed by Hamermesh and Wachter. They include examples of adjustments rejected by the court as unacceptable under the fair value standard in Delaware appraisals, even though they might be considered by a financial buyer under the third-party sale value standard.

1. Usurped corporate opportunities

If a corporate opportunity is wrongfully usurped prior to the transaction, the court will constructively attribute the corporate opportunity to the corporation and adjust fair value to reflect this misconduct. Misappropriation of a corporate opportunity by a control shareholder has been addressed in appraisal cases when the misconduct was not known to the dissenters until after the transaction that triggered the appraisal.

In its seminal 1989 *Cavalier* decision, the Supreme Court discussed a diversion of assets to a related company and stated:

⁹³Hamermesh and Wachter, "Cornfields" at 159.

The . . . corporate opportunity claim, if considered on its derivative merits, would inure almost entirely to the benefit of the alleged wrongdoers, an inequitable result at variance with the fair value quest of the appraisal proceeding. . . . [T]he Vice Chancellor found that [petitioner] did not have knowledge of the basis for the corporate opportunity claim prior to the institution of the appraisal proceeding and that, as a matter of credibility, those claims were based on misrepresentations by the principal shareholders. We conclude that, under the unusual configuration of facts present here, the corporate opportunity claim was assertable in the [appraisal] proceeding.⁹⁴

ONTI v. Integra Bank also involved an abuse of corporate opportunity. Within days of closing the squeeze-out cash merger, the control shareholder merged his company with a publicly traded company. Plaintiffs asked that the appraisal valuation take into consideration their pro rata portion of the market value of shares that the defendant received in the later merger. The court ruled in favor of the plaintiffs, stating, "I think it is clear that it is 'not the product of speculation' that the [subsequent] Transaction was effectively in place at the time of the Cash-Out Mergers."⁹⁵

2. Improper benefits to control shareholder

The court may make adjustments to eliminate the agency costs of improper actions by a control party that were not known to shareholders before the transaction and that affect current and future cash flow. For example, in *ONTI*, the court adjusted the projection underlying the DCF calculation by doubling the fees receivable from an affiliate of the controller, which had been paying less than the contractual rate.⁹⁶

In another case, the court accepted adjustments to eliminate the adverse consequences of the abusive actions of the Controller. The court adjusted for "excessive management fees, an unexplained inter-company loan, an unexplained corporate allocation, and an overcharge by a vendor"⁹⁷ as well as "the sale and leaseback of Montgomery's cell sites and towers," which "was clearly an inappropriate exaction by [the controller] due to its corporate control."⁹⁸

The court may also make adjustments when it can be shown that payments to inside shareholders were not for services rendered. In 1991 the court accepted a DCF

⁹⁴*Cavalier* at 1143–1144.

⁹⁵*ONTI, Inc. v. Integra Bank*, 751 A.2d 904, 930 (Del. Ch. 1999).

⁹⁶*Id.* at 910.

⁹⁷*Dobler v. Montgomery Cellular Holding Co.*, 2004 Del. Ch. LEXIS 139 (Oct. 4, 2004) at *69; *aff'd in part, rev'd in part on other grounds, Montgomery Cellular v. Dobler*, 880 A.2d 206 (Del. 2005). The Court of Chancery stated, "The management fee charged by [parent] can be reasonably interpreted to be a corporate charade by which the parent removed money from its subsidiary."

⁹⁸*Id.* at *71.

analysis in which officers' salaries were adjusted to exclude a portion that, because compensation was proportional to equity ownership, was deemed to be a return on equity.⁹⁹ That decision was cited in a 2011 decision where the court pointed out that "Delaware law on fair value . . . empower[s] a court to make normalizing adjustments to account for expenses that reflect controller self-dealing when the plaintiff/petitioner provides an adequate evidentiary basis for the adjustment."¹⁰⁰

Claims relating to the control shareholder's improper conduct that were known (and thus could have been challenged) prior to a squeeze-out have been excluded from consideration in Delaware appraisals. For example, in two cases where the petitioners claimed that improperly issued shares had diluted their interests, the court determined that it could not address these claims in an appraisal context.¹⁰¹

3. Improvements not dependent on merger

The court considers future events that were not speculative as part of going-concern value under the fair value standard. Hamermesh and Wachter summarize this concept:

In fact, we believe that both finance theory and Delaware case law are consistent with our view that minority shareholders have a right to "fair value" that incorporates not only current assets *but also future reinvestment opportunities, so long as those reinvestment opportunities reflect pre-merger plans or policies of the corporation and its controlling shareholder* [emphasis added]. . . . These reinvestment opportunities will not have been taken at the time of the merger, because they are to be funded with future free cash flow. Consequently, the assets purchased as part of the reinvestment opportunities will not exist at the time of the merger. However, these assets are as much a part of the present value of the corporation as are the value of the existing assets.¹⁰²

Consistent with this view and with customary valuation practice, income should be normalized so that nonrecurring items should be excluded from valuation calculations. The Court of Chancery has faulted an expert for not normalizing earnings data, pointing out, "The earnings figures used to derive the earnings base should be adjusted to eliminate non-recurring gains and losses."¹⁰³

Normalizing adjustments include not only items classed as "extraordinary" under GAAP, but also nonrecurrent items in the income account such as gains

or losses from litigation and (if truly nonrecurrent) restructuring costs. The normalizing adjustments accepted in Delaware include the adjustments described in Chris Mercer's "*Business Valuation: An Integrated Theory*" as "Type 1 Normalizing Adjustments": "These adjustments eliminate one-time gains and losses, other unusual items, discontinued business operations, expenses of non-operating assets, and the like. . . . [T]here is virtually universal acceptance that Type 1 Normalizing Adjustments are appropriate."¹⁰⁴

The cost of reinvestment opportunities should be taken into account. The court decided in a 2005 appraisal that it was not speculative to consider the cost of a cellular telephone company's prospective conversion of its network to higher future industry standards. It ruled that the expert "should have incorporated the effects of this expected capital improvement in his projections."¹⁰⁵

H. Fair value is not third-party sale value

Fair value in Delaware is not hypothetical third-party sale value. The Delaware Supreme Court wrote in 2010: "Importantly, this Court has defined 'fair value' as the value to a stockholder of the firm as a going concern, *as opposed to the firm's value in the context of [a strategic] acquisition or other transaction* [emphasis added]."¹⁰⁶

For this reason, some normalizing adjustments that an analyst might normally consider in performing a valuation are rejected in Delaware appraisals. These are the adjustments that Mercer describes as "Type 2 Normalizing Adjustments":

These adjustments normalize officer/owner compensation and other discretionary expense that would not exist in a reasonably well-run, publicly traded company. Type 2 Normalizing Adjustments should not be confused with the control adjustments or Type 1 Normalizing Adjustments.¹⁰⁷

The position of the Delaware courts is that a company should be valued as it is being run and that such adjustments as normalizing officer/owner compensation would reflect third-party sale value. In addition, pro forma adjustments that Mercer describes as "Financial Control Adjustments" and "Strategic Control Adjustments"¹⁰⁸ clearly reflect value to an acquiror and therefore are rejected in Delaware appraisals.

In 1997 the Delaware Supreme Court rejected petitioner's claim that earnings and projections should be adjusted because the control shareholder had been and

⁹⁹*Radiology Associates*, 611 A.2d 485, 491–492.

¹⁰⁰*Reis* at 472.

¹⁰¹*Cavalier* at 1146; *Gentile v. SinglePoint Financial*, 2003 Del. Ch. LEXIS 21 (Del. Ch. Mar. 5, 2003) at *17–*21.

¹⁰²Hamermesh and Wachter, "Cornfields" at 158.

¹⁰³*Reis* at 470.

¹⁰⁴Z. Christopher Mercer and Travis W. Harms, *Business Valuation: An Integrated Theory*, 2d ed. (New York; John Wiley & Sons, 2007), p. 113.

¹⁰⁵*U. S. Cellular*, 2005 Del. Ch. LEXIS 1 at *56.

¹⁰⁶*Golden Telecom*, 11 A.3d 214 at 217.

¹⁰⁷*Mercer & Harms* at 113.

¹⁰⁸*Id.* at 117–120.

continued to be materially overpaid. It noted that there was no plan prior to the merger to adjust that compensation. The court ruled that “in the absence of a derivative claim attacking excessive compensation, the underlying issue of whether such costs may be adjusted may not be considered in an appraisal proceeding.”¹⁰⁹ It concluded that “going business value of the corporation at the moment before the merger . . . does not include the capitalized value of possible changes which may be made by new management [emphasis added].”¹¹⁰

A 2011 decision rejected an adjustment to earnings premised on the assertion that that the company was overspending on research. The Court of Chancery ruled, “Because a reduction in R&D expense only could be made by a new controller of Hazelett Strip-Casting, adjustments to reflect those changes would generate a third-party sale value, not going concern value.”¹¹¹ The court stated that its conclusion was based on “the well-established principle of Delaware law that minority shareholders have no legal right to demand that the controlling shareholder achieve—and that they be paid—the value that might be obtained in a hypothetical third-party sale.”¹¹²

Similarly, the Court of Chancery declined in *ISN Software* (2016) to make adjustments for challenged expenditures that it did not deem to be wasteful:

I do not make separate adjustments for executive compensation, charitable contributions, or private jet usage. Those expenditures were a part of the Company’s operative reality on the date of the Merger, and there is no evidence sufficient, in my opinion, to demonstrate that they represent waste or actionable breaches of fiduciary duty; as such, they would have likely continued in a going-concern ISN.¹¹³

I. Taxes are considered only if they are “operative reality”

The operative reality concept has also been used to justify the exclusion of deferred taxes on investment assets (built-in capital gains) that management does not currently intend to sell. In *Paskill*, the Supreme Court ruled:

The record reflects that a sale of its appreciated investment assets was not part of Okeechobee’s operative reality on the date of the merger. Therefore, the Court of Chancery should have excluded any deduction for the speculative future tax

liabilities that were attributed by Alcoma to those untemplated sales.¹¹⁴

This differs from the Supreme Court’s ruling that accepted deferred taxes in *Technicolor* because in that case, Technicolor’s management had already decided to sell the relevant assets. The built-in gain on that asset sale was the operative reality on the date of the merger.¹¹⁵

In a 2006 decision, the Court of Chancery used the operative reality concept when it accepted taxes and other expenses paid as a result of an asset sale directly related to a merger. Carter-Wallace sold the assets of its consumer products business simultaneously with the merger into MedPointe Healthcare of its health-care business. Each transaction was contingent on the other. The asset sale resulted in substantial capital gains taxes and expenses. The petitioner unsuccessfully argued that the taxes and expenses should not be deducted in determining appraisal value because the asset sale was not completed prior to the date of the merger. The court said, “There is no principled distinction between an asset sale occurring a few hours before the merger and a sale on the day before the merger” and based its appraisal of Carter-Wallace on the company’s value after the asset sale, giving effect to all related expenses including taxes on the asset sales.¹¹⁶

A 2011 decision rejected the deduction of potential taxes and selling expenses and “add[ed] the full appraised value of the non-operating real estate” that the company had no current intent to sell.¹¹⁷ Moreover, since the company expected to utilize its net operating losses, the Court of Chancery also added the potential tax benefit of the carryforward:

Hazelett Strip-Casting has a history of generating taxable earnings, and the capitalized earnings valuation anticipates that it will continue doing so in a manner that will enable the Hazelett family to take advantage of the NOL. I therefore add \$258,000, representing the full value of the NOL.¹¹⁸

J. Tax-affecting S corporations

Since a company is valued in a Delaware appraisal as it is being run by its current management, not as it might be run by the buyer, a C corporation is valued based on C corporation taxes even if the buyer intends to convert it to an S corporation, and an S corporation is valued inclusive

¹⁰⁹*Gonsalves v. Straight Arrow Publishers*, 701 A.2d 357, 363 (Del. 1997).

¹¹⁰*Id.*

¹¹¹*Reis* at 471.

¹¹²*Id.*, quoting Hamermesh and Wachter, “Cornfields” at 154.

¹¹³*In re ISN Software Corp. Appraisal Litig.*, 2016 Del. Ch. LEXIS 125 (Aug. 11, 2016) at *17, fn. 46.

¹¹⁴*Paskill Corp. v. Alcoma Corp.*, 747 A.2d 549, 552 (Del. 2000).

¹¹⁵*Technicolor* 1996 at 298.

¹¹⁶*Cede & Co., Inc. v. MedPointe Healthcare*, 2004 Del. Ch. LEXIS 124 (Aug. 16, 2004) at *29 (“The inquiry here is not one of hours, but of whether one two-step transaction, with all components occurring in a certain order and substantially simultaneously, may (or must) be divided for valuation purposes.”).

¹¹⁷*Reis* at 476.

¹¹⁸*Id.*

Table 1
Court’s Calculation from Delaware Open MRI¹²¹

	C Corporation	S Corporation	S Corporation Valuation
Income before Tax	\$100	\$100	\$100
Corporate Tax Rate	40%	–	29.4%
Available Earnings	\$60	\$100	\$70.60
Dividend or Personal Income Tax Rate	15%	40%	15%
Available after Dividends	\$51	\$60	\$60

of the tax benefits of being an S corporation. In Delaware Open MRI (2006), then-Vice Chancellor Leo Strine, Jr. (now Chief Justice of the Delaware Supreme Court) explained that “an S corporation structure can produce a material increase in economic value for a stockholder and should be given weight in a proper valuation of the stockholder’s interest.”¹¹⁹ He ruled: “[W]hen minority stockholders have been forcibly denied the future benefits of S corporation status, they should receive compensation for those expected benefits and not an artificially discounted value that disregards the favorable tax treatment available to them.”¹²⁰

He determined the implied effective S corporation tax rate in a 2006 appraisal case. Using a 40% corporate tax rate, a 40% personal tax rate, and a 15% tax on S corporation dividends, he calculated the implied pro forma S corporation tax rate at the corporate level that would give shareholders the same after-tax earnings that C corporation shareholders would receive after both corporate taxes and taxes on corporate dividends to be 29.4% (Table 1).

The 29.4% effective pro forma S corporation tax rate was calculated by taking the amount available to S corporation shareholders, grossing it pro forma for a 15% dividend tax, and then determining the implied effective tax rate.

Chancellor Andre Bouchard applied the same method in a C corporation appraisal in 2015.¹²² He used the dissenter’s “actual tax rates as a Maine resident” and calculated the taxes applicable to C corporation dividends as 31.75%: “the sum of the 20% federal tax on dividends, the 3.8% Net Income Investment Tax (NIIT) imposed by the Affordable Care Act, and the 7.95% Maine state tax on dividends.”¹²³

The Strine formula can be expressed in a mathematical formula that can be applied using different tax rates. If P

is the relevant marginal personal tax rate, D is the tax rate applicable to C corporation dividends, and E is implied effective tax rate on the S corporation, then

$$E = 1 - (1 - P)/D.$$

When applying this formula, the valuator should consider the impact of state personal income taxes. Interestingly, the formula is independent of the C corporation tax rate.

III. Valuations in Other Jurisdictions May Differ from Delaware

A. Appraisals under the MBCA could be higher than in Delaware

The Delaware statute excludes appreciation “arising from the accomplishment or expectation” of the transaction without the “inequitable” qualification that had been included in the pre-1999 MBCA. The Official Comments to the 1999 MBCA point out: “[T]he exclusionary clause in the prior Model Act definition, including the qualification for cases where the exclusion would be inequitable, has been deleted. Those provisions have not been susceptible to meaningful judicial interpretation.”¹²⁴

The Official Comments further state:

Customary valuation concepts and techniques will typically take into account numerous relevant factors, including assigning a higher valuation to corporate assets that would be more productive if acquired in a comparable transaction but *excluding any element of value attributable to the unique synergies of the actual purchaser* [emphasis added].

For example, if the corporation’s assets include undeveloped real estate . . . the court should consider the value that would be attributed to the real estate . . . in a comparable transaction. The court should not, however, assign any additional value based upon the specific plans or special use of the actual purchaser.¹²⁵

The revised MBCA is thus more expansive than Delaware in that it includes in fair value *any* improvement

¹¹⁹Delaware Open MRI at 327.

¹²⁰Id. at 328

¹²¹Id. at 328.

¹²²Owen v. Cannon, 2015 Del. Ch. LEXIS 165 (June 17, 2015) at *72.

¹²³Id. at *71.

¹²⁴Official Comments to MBCA, §13.01 (1999).

¹²⁵Id.

that could be made without unique input from a third-party acquiror. While Delaware includes any improvements that are *planned or contemplated* by existing management, it excludes “the capitalized value of possible changes which may be made by new management.”¹²⁶ In contrast to Delaware, the 1999 MBCA would determine fair value adjusted for business opportunities which management *has not yet planned to exploit* or for excessive compensation paid to management. Thus, states that adopt the language in the 1999 MBCA could accept Mercer’s “Type 2 Normalizing Adjustments” that Delaware rejects.¹²⁷

B. Some states consider financial control value

Most states follow the Delaware interpretation of fair value, valuing a company as it exists at the valuation date. The case law of some states, however, defines going-concern value more expansively. In 1986, the Supreme Judicial Court of Massachusetts ruled:

As a going concern, the value of an enterprise such as the Old Patriots is *the price a knowledgeable buyer would pay for the entire corporation*, including the National Football League (NFL) franchise, the stadium lease, various contracts, goodwill, and other assets and liabilities [*emphasis added*].¹²⁸

Three years later, in the widely cited *McLoon* case, the Supreme Judicial Court of Maine concluded:

Especially in fixing the appraisal remedy in a closely held corporation, the relevant inquiry is what is the *highest price a single buyer would reasonably pay for the whole enterprise*, not what a willing buyer and a willing seller would bargain out as the sales price of a dissenting shareholder’s share in a hypothetical market transaction [*emphasis added*].¹²⁹

New York similarly defines “fair value” as the amount that would be paid by an arm’s length non-synergistic buyer, that is, financial control value:

[I]n fixing fair value, courts should determine the minority shareholder’s proportionate interest in the going concern value of the *corporation as a whole*, that is, “what a willing purchaser, in an arm’s length transaction, would offer for corporation as an operating business” [*emphasis in original*].¹³⁰

¹²⁶*Gonsalves v. Straight Arrow*, 701 A.2d 357, 363.

¹²⁷See “Fair value is not third-party sale value” above.

¹²⁸*Sarrouf v. New England Patriots Football Club, Inc.*, 492 N.E.2d 1122, 1125 (Mass. 1986).

¹²⁹*In re Val. of Common Stock of McLoon Oil Co.*, 565 A.2d 997, 1004 (Me. 1989) (“*McLoon*”).

¹³⁰*Friedman v. Beway Realty Corp.*, 87 N.Y.2d 161, 168 (N.Y. 1995) (“*Beway*”), quoting *Matter of Blake v. Blake Agency*, 107 A.D.2d 139,146 (N.Y. App. 1985) (“*Blake*”).

In the recent *AriZona Beverages* case, the New York trial court stated:

[The Court] value[d] AriZona using the “financial control” measurement, that is, “the value of a company exposed to a representative group of buyers who are not expecting synergies, who are looking at the value of the business on a standalone basis, who may not be able to run the company a little differently . . . a little better but not differently like the synergies.”¹³¹

Other states where decisions have cited the above language from *McLoon* include Connecticut,¹³² Florida,¹³³ Indiana,¹³⁴ Iowa,¹³⁵ Missouri,¹³⁶ Vermont,¹³⁷ and Virginia.¹³⁸ In these states that define fair value as financial control value, nonsynergistic changes that a new management might undertake would be relevant to an appraisal. Thus, normalizing for these potential changes, described by Mercer as Type 2 Normalizing Adjustments,¹³⁹ would be generally be accepted in those states, as would the use of a normalized capital structure rather than a company’s actual capital structure. These adjustments would have a positive impact on valuations in most situations.

However, a financial control standard could have a negative effect on valuations of flow-through entities such as S corporations. If the likely acquiror of a C corporation is an S corporation, the appropriate pro forma tax rate for valuation purposes would be a C corporation tax rate. When the standard is going-concern value as the company is being run, the valuation includes the tax benefits of an S corporation, as was done in *Delaware Open MRI*.¹⁴⁰

Similarly, a court using a financial control standard could arrive at a lower value than a Delaware court when a company has significant built-in capital gains on marketable assets. Delaware does not accept deductions for taxes on built-in gains unless there is an intent to sell the assets,¹⁴¹ but a financial control buyer would be

¹³¹*Ferolito v. AriZona Beverages USA LLC*, 2014 N.Y. Misc. LEXIS 4709 (N.Y. Supr., Oct. 14, 2014) at *19–*20, quoting Chris Mercer’s trial testimony.

¹³²*Devivo v. Devivo*, 2001 Conn. Super. LEXIS 1285 (May 8, 2001) at *16.

¹³³*G & G Fashion Design, Inc. v. Garcia*, 870 So. 2d 870, 872 (Fla. App. 2004).

¹³⁴*Lees Inns of America, Inc. v. Lee*, 924 N.E.2d 143, 156 (Ind. App. 2010).

¹³⁵*Northwest Investment Corp. v. Wallace*, 741 N.W.2d 782,791 (Iowa 2007).

¹³⁶*Swope v. Siegel-Robert, Inc.*, 243 F.3d 486, 492 (8th Cir. 2001).

¹³⁷*In re 75,629 Shares of Common Stock of Trapp Family Lodge, Inc.*, 725 A.2d 927, 931 (Vt. 1999).

¹³⁸*U.S. Inspect, Inc. v. McGreevy*, 57 Va. Cir. 511, 526, 2000 Va. Cir. LEXIS 524, at *33–*34 (Nov. 7, 2000).

¹³⁹Mercer & Harms at 113.

¹⁴⁰*Delaware Open MRI* at 330.

¹⁴¹See “Taxes are considered only if they are ‘operative reality’” above.

highly likely to bid a lower price for such a company than for an otherwise identical company whose marketable assets had a higher tax basis.

C. Appraisals by the Controller of the Currency

National banks are incorporated under federal law. Shareholders dissenting from mergers of national banks or conversion of national banks into state banks are not entitled to a judicial appraisal, but instead are granted the right to an appraisal by the majority of a three-person panel.¹⁴² The panel consists of one person chosen by holders of a majority of the dissenting shares, one chosen by the bank, and a third person chosen by the first two. Either party may appeal and ask the Controller of the Currency for a binding reappraisal.¹⁴³ However, in stock-for-stock mergers, the shares that would have been

delivered to the dissenters must be sold in a public auction. If the auction price is higher than the appraised value, the dissenters are entitled to the higher price.¹⁴⁴

This article will be continued in the next issue of *Business Valuation Review*.

This article is materially revised and updated from Chapter 3 in *Standards of Value: Theory and Application*, 2nd ed. (New York: Wiley, 2013) and is published with permission of John Wiley & Sons, Inc. Michelle Patterson, JD, PhD, assisted with the conceptualization, organization, and writing.

¹⁴²12 U.S.C. § 214a, § 215, § 215a.

¹⁴³Valuations by the OCC use the Delaware block method and do not use DCF. (OCC, *Business Combinations*, Dec. 2006, pp. 39–40, available at <http://www.occ.gov/publications/publications-by-type/licensing-manuals/bizcombo.pdf>).

¹⁴⁴12 U.S.C § 215(d), § 215a(d).

Comparing Three Convertible Debt Valuation Models

Dwight Grant, PhD

In this article, I (a) describe and illustrate the implementation of three convertible debt valuation models, (b) show how their values for convertible debt respond to changes in the underlying valuation parameters, (c) examine the effects of changing each of the models such that the credit spread and the probability of default are not constant but vary inversely with the stock price, and (d) measure and compare the accuracy of each model when it is calibrated to convertible debt issuance prices and then used to forecast the convertible debt price one year later.

Introduction

Appraisers frequently value nontraded convertible debt, either to estimate the fair value of the debt component of the convertible or to estimate the value of one or more derivatives embedded in the convertible debt. There is an extensive literature on the valuation of convertible debt, and many models have been proposed. In my experience, appraisers most frequently use a credit spread model developed by Tsiveriotis and Fernandes when they were at Morgan Stanley (the “TF” model¹). Less frequently, appraisers use a probability-weighted discount rate model developed by Bardhan et al. when they were at Goldman Sachs & Company (the “GS” model²). These two models were developed independently in the same time period as practical approaches to the valuation of a complex hybrid security. The TF model figures much more prominently in the literature, having been cited 295 times as compared to 28 times for the GS model. More recently, appraisers have begun to use “jump-to-default” models. A paper by Milanov and Kounchev (the “MK” model³) describes one good example of such a model.

The TF model values the conversion feature in an option-pricing risk-neutral framework and values the principal and interest payments in a “real-world” discounted cash flow framework. Consequently, the model discounts conversion

values at the risk-free rate and principal and interest payments at a credit-adjusted rate. The GS model differs in that it discounts all cash flows by a weighted average of the risk-free rate and the credit-adjusted rate, with the weights determined by the probabilities of conversion to common stock and redemption. Both models implement the calculation of cash flows and their discounting with sets of lattices as described later herein. The foundation for these lattices is a common stock price lattice.⁴ In contrast, the MK model values all payments in a risk-neutral framework. The foundation of this model is a CRR stock price lattice that adds a jump to default at each date. Necessarily, this model is augmented with assumptions about recovery rates in the event of default.

The purpose of this article is to demonstrate the implementation of the models, illustrate how they respond to changes in inputs, and compare their performance. My perspective on performance is that of an appraiser who is pricing nontraded convertible debt by calibrating a model to the issuance price and is concerned with the performance of the developed model at a later date. This contrasts with previous empirical studies that examined the performance of models in pricing traded convertible debt based on inputs derived independently of the convertible debt trading price.⁵

I first describe the mechanics of each model and illustrate each with a numerical example. Second, I compare how the values of the three models vary with changes in the valuation parameters. Third, I examine the effects of changing each of the models such that the credit spread and the probability of default are not constant but vary inversely with the stock price. Put another way, I introduce a credit spread lattice and a probability of default lattice instead of using constant values for all

Dwight Grant is a managing director in PwC’s Value Analytics and Derivatives Practice. He is based in San Francisco, California. He thanks Dmitry Kosorukov, Xinajian Zou, Yusu Ziang, and especially Seema Ranka for their assistance.

¹Tsiveriotis and Fernandes (1998).

²Bardhan et al. (1994).

³Milanov and Kounchev (2012). Hull (2011) presents a similar model. See also de Spiegeleer and Schoutens (2011).

⁴The TF model uses a Cox, Ross, and Rubinstein (1979) lattice, while the GS model uses a Jarrow and Rudd (1983) model. To facilitate comparisons, we employ the CRR lattice for the GS model.

⁵See Zabolotnyuk, Jones, and Veld (2010).

future stock prices. While this is conceptually an appealing improvement for each model, the illustrations presented here suggest that the improvement in the model is likely not worth the cost of the added complexity. Fourth, I use market data to provide further insight into the valuation results that each model provides. I find that the three models perform almost identically as price forecasting models. In addition, the results suggest that price forecasting is better when one continues to use the original calibrated parameters for the volatility of the underlying equity and credit risk rather than holding credit risk constant and updating the volatility.

Mechanics of the TF Model

The foundation of this model is a binomial stock price lattice. A second lattice tracks the value of the convertible debt based on its conversion value, along the paths where it is converted. A third lattice tracks the value of the convertible debt based on its interest payments and the payment of principal at maturity along the price paths where it is redeemed. A fourth lattice tracks the combined conversion and debt values. This model can be illustrated with the following example: The face value of the convertible debt is 100. It has a five-year term and a coupon rate of 3.28%, paid annually. The risk-free rate and the credit spread are 2% and 8%, respectively.⁶ The debt converts into one share of common stock with a current price of 80. The annual volatility of the common stock is 30%. For this illustration, I model one-year time steps in the lattices. See Table 1.

I use a CRR stock price lattice. At each node in the lattice, the stock price can move up by the factor u or down by the factor d . The probabilities of the up and down moves are p_u and p_d , respectively:

$$u = e^{\sigma\sqrt{\Delta t}}; d = \frac{1}{u}, p_u = \frac{e^{r\Delta t} - d}{u - d}, p_d = 1 - p_u,$$

where r is the risk-free rate, and Δt is the time between nodes. For example, Table 1 illustrates that in the first year, the stock price either moves from 80.00 up to 107.99, with probability $p_u = 46\%$, or down to 59.27, with a probability $p_d = 54\%$.

The conversion lattice is connected to the stock price lattice at the last date, with the values being the conversion value when it exceeds face value and zero otherwise. The earlier nodes are the discounted present values calculated in a backward recursive manner. In the Table 1, consideration of more complex features such as forced early conversion is excluded.

The straight debt lattice values are also calculated in a backward recursive manner, with the values at the last date being face value plus interest when the face value

exceeds the conversion value and interest otherwise. The earlier nodes are the discounted present values calculated in a backward recursive manner, with the discount rate being the risk-free rate plus a credit spread.

The total value lattice is, in this relatively simple example, the sum of the conversion and the straight debt lattices. In more complex examples, it is used to track the effects on value of features such as forced conversion.

Table 1 produces a value for the convertible that is 3.44% above par. When the model is expanded to sixty steps, the value is almost exactly par, which was the objective in choosing the inputs.

Mechanics of the GS Model

The foundation of this model is the same binomial price lattice. A second lattice tracks the probability of conversion at each node. A third lattice calculates a node-specific discount rate based on the probability of future conversion. A fourth lattice tracks the convertible bond value. This model is illustrated in Table 2 using the same parameters used to illustrate the TF model.

The probability lattice shows that entries at date 5 are either 100% or 0%, depending on the terminal stock price, because at that date, it is known whether conversion occurs. Corresponding to these results, the discount rate lattice shows the risk-free rate, 2%, for those cash flows where conversion occurs, and the credit-adjusted rate, 10%, for those where it does not occur. We illustrate the valuation of the convertible bond with these examples:

$$\begin{aligned} 152.26 &= p_u 200.50(1.01) - 2 + p_d 111.27(1.01) - 2 \\ 104.02 &= p_u 111.27(1.01) - 2 + p_d 103.28(1.05) - 2 \\ 96.96 &= p_u 103.28(1.05) - 2 + p_d 103.28(1.05) - 2, \text{ and} \\ 95.71 &= p_u 104.02(1 + .5(6.33\%)) \\ &\quad - 2 + p_d 96.96(1.05) - 2, \text{ where} \\ 6.33\% &= (46\%)(2\%) + (1 - 46\%)(10\%) \end{aligned}$$

Note that the discount rate, 6.33%, is a weighted average of the risk-free rate and the risky rate, with the weights being taken from the probability of the conversion lattice. At this particular valuation node, the probability of conversion at the next time period (date) is 46%, which is the probability of the stock price moving up from 80 to 107.99, at which point conversion would occur. Likewise, the probability of conversion of 71% at date 3 is equal to $46\%(100\%) + 46\%(1 - 46\%)$.

The convertible bond value in this example is 106.06, and when the model is expanded to sixty steps, it is 97.59, i.e., 2.41% lower than that calculated in the TF model.⁷

⁷Lattices with this few steps do not provide precise estimates of value. Also, prior expectation was that with the same parameters, the TF and GS models would produce a similar but not identical convertible debt value. For these parameters, they differ by 2.41%.

⁶All interest rates are compounded semi-annually.

Table 1
The Four Lattices in the TF Model

Date	0	1	2	3	4	5	0	1	2	3	4	5
Stock price lattice						Conversion value lattice						
80.00	107.99	145.77	196.77	265.61	358.54	55.54	89.47	136.91	196.77	265.61	358.54	
	59.27	80.00	107.99	145.77	196.77		28.86	52.62	91.29	145.77	196.77	
		43.90	59.27	80.00	107.99			9.81	21.82	48.54	107.99	
			32.53	43.90	59.27				0.00	0.00	0.00	
				24.10	32.53					0.00	0.00	
					17.85						0.00	
Straight debt value lattice						Total value lattice						
47.80	39.14	23.25	8.95	6.255	3.28	103.34	128.62	160.15	205.72	271.86	361.82	
	64.18	53.34	33.07	6.255	3.28		93.04	105.96	124.36	152.02	200.05	
		78.83	73.92	55.365	3.28			88.64	95.75	103.91	111.27	
			91.22	96.958	103.28				91.22	96.96	103.28	
				96.958	103.28					96.96	103.28	
					103.28						103.28	

Mechanics of the MK Model

This model incorporates debt risk by modeling default in a risk-neutral framework. At each date, the stock price can move up or down, just as in Table 1, but it can also move to a default value, which is modelled as a percentage of the value in the node from which it is moving. Therefore, at each node in the lattice, the stock price can move up by the factor *u*, down by the factor *d*, or down by the factor *b*. The probabilities of the moves are, *p_u*, *p_d*, and *p_b*. In the MK model, default is a Poisson process, with the probability of default over any time interval being $1 - e^{-\lambda\Delta t}$. This introduces three new inputs, the risk-neutral default intensity, λ , the recovery rate on the debt in the

event of default, and the response of the stock price to default, *b*. The default intensity and recovery rate relate to the credit spread in the TF model, and the size of the stock price decline in the event of default is a new parameter. In the MK risk-neutral framework, the mathematical expressions for the variables *u* and *d* are the same as the TF model, but the probabilities are different.

$$u = e^{\sigma\sqrt{\Delta t}}; d = \frac{1}{u}, p_u = \frac{e^{r\Delta t} - de^{-\lambda\Delta t} - b(1 - e^{-\lambda\Delta t})}{u - d},$$

$$p_d = 1 - p_u - p_b, p_b = 1 - e^{-\lambda\Delta t}.$$

Table 2
The Four Lattices in the GS Model

Date	0	1	2	3	4	5	Date	0	1	2	3	4	5
Stock price lattice						Weighted-average discount rate lattice (%)							
80.00	107.99	145.77	196.77	265.61	358.54			3.87	2.20	2.00	2.00	2.00	2.00
	59.27	80.00	107.99	145.77	196.77			9.09	6.62	3.27	2.00	2.00	2.00
		43.90	59.27	80.00	107.99				9.93	9.23	6.33	2.00	
			32.53	43.90	59.27					10.00	10.00	10.00	
				24.10	32.53						10.00	10.00	
					17.85							10.00	
Probability of conversion lattice (%)						Convertible debt value lattice							
42	62	84	100	100	100		106.06	129.45	161.40	206.42	272.10	361.82	
	25	44	71	100	100			92.15	105.96	124.64	152.26	200.05	
		10	21	46	100				88.18	95.71	104.02	111.27	
			0	0	0					91.22	96.96	103.28	
				0	0						96.96	103.28	
					0							103.28	

Table 3
The Two Lattices in the MK Model

Date	0	1	2	3	4	5	0	1	2	3	4	5
	Stock price lattice						Convertible debt value lattice					
	80.00	107.99	145.77	196.77	265.61	358.54	101.30	125.74	157.79	205.18	271.67	361.82
		59.27	80.00	107.99	145.77	196.77		90.73	102.90	121.75	151.83	200.05
			43.90	59.27	80.00	107.99			86.16	92.39	100.46	111.27
				32.53	43.90	59.27				90.01	96.10	103.28
					24.10	32.53					96.10	103.28
						17.85						103.28
		24.00	32.40	43.73	59.03	79.68						
			17.78	24.00	32.40	43.73						
				13.17	17.78	24.00						
					9.76	13.17						
						7.23						

The stock price lattice in Table 3 is built with the same inputs as those used for Table 1 and the addition of a default intensity of 14.6%, a recovery rate of 40%, and a jump to default of $-70%$, meaning $b = 0.30$. To make Tables 1 and 2 comparable, the default intensity is based on the risk-free rate and the credit spread used in the TF model; namely, it is the risk-neutral default intensity consistent with a five-year, 3.28% coupon debt with a 40% recovery rate that is priced to yield 10% ($2% + 8%$). In Table 3, the stock price moves from 80.00 either up to 107.99 or down to 59.27 or into bankruptcy with a value of $24.00 = 0.30(80)$.⁸ Likewise, at date 1, if the stock price moved down to 59.27, it can next move up to 80.00, down to 43.90, or into bankruptcy at $17.78 = 0.30(59.27)$. In the bankruptcy states, the value of the convertible debt is the higher of its conversion value or the recovery value, which is 40.

Because all of the discounting in this model is at the risk-free rate, the model requires only one other lattice, one that tracks the value of the convertible debt. At date 5, the value of the convertible debt is the higher of its value in conversion or its face value, plus interest. At each of the other nodes, the value of the convertible is calculated in a backward recursive manner as the discounted probability-weighted values it can assume at the next date. In this illustration, the value of the convertible debt is 101.30, and for the sixty-step version of the model, the value is 102.94, i.e., 2.94% higher than the value produced by the TF model.

⁸This feature tends to offset the undesirable effect of the probability of default being unchanged as the stock price increases.

Comparison of the Changes in Values of the Three Models as Parameters Change

Having illustrated the mechanics of the models, I now compare the changes in their values when their parameters are varied. To increase the precision of the results discussed here, they are based on sixty-step implementations of each model. I adopt the parameters discussed in the previous section as a base case. I measure the effect on the price of the convertible for two additional credit spreads, 4% and 12%, for levels of price decline $-70%$ and $-100%$ ($b = 0.30$ and $b = 0.0$) and for two tenors, five years and ten years. For the ten-year tenor, the coupon rate is decreased to 2.93%, so that the TF model calibrates to par. Table 4 presents the results.

I discuss the results for the TF and GS models first. For both the five-year term and the ten-year term, the base case value of the convertible debt is par for TS by design and close to par for the other two models. For each term, I increase and decrease the credit spread by 400 basis points. As would be expected, the value of the convertible increases when the credit spread declines and decreases when the credit spread increases. For these parameters, the changes in values are 10.8% and $-8.6%$ for the five-year term and 15.0% and $-10.4%$ for the ten-year term. For the GS models, the corresponding changes in value are similar, but somewhat larger in both directions, 11.6% and $-10.0%$ for the five-year term and 17.8% and $-14.0%$ for the ten-year term.

The MK model values the convertible at very close to par for the base case when the equity jump is $-70%$, i.e., 102.94 and 100.60 for the five-year and ten-year terms, respectively.

Table 4
Convertible Debt Values for the TF and MK Models for Combinations of Parameters

<i>S/X</i> = 80/100	80%				80%			
Term (years)	5.00				10.00			
Coupon Rate (%)	3.28				2.93			
Model	TF	GS	MK		TF	GS	MK	
Recovery Rate (%)			40.00				40.00	
Credit Spread/Default Intensity (%)	4.00	4.00	8.36		4.00	4.00	7.27	
Equity Jump (%)			-70	-100			-70	-100
Volatility (%)	30.0	30.0	35.1	43.7	30.0	30.0	35.4	44.5
Pure Bond	88.40	88.40	88.40	88.40	77.15	77.15	77.15	77.15
Conversion Value	<u>22.18</u>	<u>21.26</u>	<u>22.93</u>	<u>27.86</u>	<u>37.84</u>	<u>35.60</u>	<u>36.75</u>	<u>47.22</u>
Convertible	110.58	109.66	111.33	116.26	114.98	112.75	113.90	124.37
Credit Spread/Default Intensity (%)	8.00	8.00	14.60		8.00	8.00	18.57	
Equity Jump (%)			-70	-100			-70	-100
Volatility (%)	30.0	30.0	40.2	55.5	30.0	30.0	42.5	60.6
Pure Bond	74.05	74.05	74.05	74.05	55.93	55.93	55.93	55.93
Conversion Value	<u>25.96</u>	<u>23.63</u>	<u>28.89</u>	<u>40.54</u>	<u>44.07</u>	<u>38.12</u>	<u>44.67</u>	<u>67.91</u>
Convertible	100.00	97.68	102.94	114.59	100.00	94.05	100.60	123.85
Credit Spread/Default Intensity (%)	12.00	12.00	57.28		12.00	12.00	93.56	
Equity Jump (%)			-70	-100			-70	-100
Volatility (%)	30.0	30.0	45.7	67.2	30.0	30.0	74.1	121.8
Pure Bond	62.35	62.35	62.35	62.35	41.35	41.35	41.35	41.35
Conversion Value	<u>29.02</u>	<u>25.07</u>	<u>35.08</u>	<u>54.22</u>	<u>48.24</u>	<u>38.17</u>	<u>47.17</u>	<u>80.56</u>
Convertible	91.37	87.42	97.44	116.57	89.59	79.52	88.52	121.91

However, if an equity jump of -100% is selected, the value of the convertible is substantially larger, 114.59 and 123.85 for the five-year and ten-year terms, respectively. It is important to understand this counterintuitive result. The total volatility of the common stock price is a function of the familiar volatility used in the TF model, $\sigma=30\%$ in this example, and the size of the equity jump and the probability of default. Specifically, total stock price volatility for the MK model⁹ is $\sqrt{\sigma^2 + \lambda b^2}$. When b is 0.30, total volatility is 42.5%, and when b is 0.00, total volatility is 55.5%. This increase in volatility explains why the value of the convertible debt increases when the equity price jump is -100% rather than -70% . This point also highlights that the two models are not comparable with respect to volatility. For the MK model to have a total volatility in the base case with a stock price jump of -70% , the usual volatility term, referred to as σ , would have to be 13.6%. For those parameters, the value of the convertible would decrease from 102.94 to 96.48. I will return to this subject when I discuss the estimation of the parameters for the models and the models' relative merits.

When the credit spread is decreased by 400 basis points, the corresponding default intensity decreases from 14.60% to 8.36%; when the credit spread is increased by 400 basis points, the default intensity increases from 14.60% to 57.28%. The effect on the value of the convertible is not as

⁹See Milanov and Kounchev (2012), who derive this relationship based on the assumption that default and stock price are uncorrelated.

straightforward as for the TF and GS models and depends on the size of the equity price jump because of its effect on volatility, as just discussed. I highlight one case: When the equity jump is -100% , a decrease and an increase in the credit spread of 400 basis points increases the value of the convertible debt by almost equal amounts, 1.67 and 1.98. The increase in the value of the convertible in the MK model in response to an increase in the credit spread is a clear and important difference between the MK model and the TF and GS models, and I view it as a negative feature of the MK model.

The values produced by the models can also be compared for different degrees of moneyness. Table 5 reports the values of the convertible debt for the five-year and ten-year terms, the two equity jumps, -70% and -100% , and three levels of moneyness, 60%, 80% (the base case), and 100%. The changes in values attributable to the change in moneyness (common stock price) are both reasonable and consistent among the three models.

Addressing a Common Shortcoming of the Models

All three of these models implicitly assume that the probability of bankruptcy is independent of the price of the underlying common stock price. In general, that is an unreasonable assumption. I would expect the credit spread in the TF and the GS models and the default intensity in

Table 5
Convertible Debt Values for the TF and MK Models for Degrees of Moneyness

Term	5.00				10.00			
Coupon Rate (%)	3.28				2.93			
Model	TF	GS	MK		TF	GS	MK	
Recovery Rate (%)			40.00				40.00	
Credit Spread/Default Intensity (%)	8.00	8.00	14.60		8.00	8.00	18.57	
Equity Jump (%)			-70	-100			-70	-100
Volatility (%)	30.0	30.0	40.2	55.5	30.0	30.0	42.5	60.6
S/X	<u>Convertible Value</u>				<u>Convertible Value</u>			
60/100 = 60%	87.99	86.79	89.54	97.53	80.29	75.32	85.55	104.03
80/100 = 80%	100.01	97.68	102.94	114.59	100.00	94.05	101.04	123.77
100/100 = 100%	115.10	111.83	118.45	132.96	114.23	104.93	117.83	143.64
For S/X = 60%: % Change from 80%	-12.0	-11.1	-13.0	-14.9	-19.7	-19.9	-15.3	-15.9
For S/X = 100%: % Change from 80%	15.1	14.5	15.1	16.0	14.2	11.6	16.6	16.1

the MK model to be inversely related to the price of the common stock. In this section of the paper, I examine the effects on value of introducing this relationship.

The results for the TF and GS models are similar, and so I discuss the TF model in detail. The credit spread is expressed as a modified power function with the degree of moneyness as the independent variable. This produces a credit spread lattice in which the credit spread at each node is determined by the power function and the stock price at the corresponding node in the stock price lattice. Credit adjustment of debt cash flows is node (stock price) specific. I identify parameters of the modified power function such that the credit spread for a stock price of 80 is approximately the 8% used in the base case. Figure 1 displays the function.

For the MK model, I follow a similar process. In this case, I develop a default intensity lattice that is a function of the stock price. This gives rise to corresponding probability lattices for the each of the three possible price moves, *u*, *d*, and *b*, and the default intensity and the probabilities are all node (stock price) specific. I identify a function such that the default intensity for a price of 80 is approximately the 14.6% used in the base case. Figure 2 displays the function.¹⁰

The operative question is whether these changes in the models, which are complex and require judgment with respect to the functional form of the credit spread and default intensity equations, are worth the effort. The results in Table 6 suggest that they are not. For the five-year convertible debt, the price differences caused by the more complex models range from less than 1% to a maximum of 4%. To illustrate this result, consider the base case. With a constant credit spread or default intensity, the values of the five-year debt are 100.00,

102.94, and 114.59 for the TF model and the two versions of the MK model, respectively. When the price-varying spread and default intensity are introduced, the values are 98.02, 103.79, and 111.62, respectively, i.e., differences of only 2.0%, -0.8%, and 2.6%.

Examining the Pricing Performance of the Three Models

I collected data on convertible debt issued by companies with publicly traded common stock from

Table 6
Convertible Debt Values for the TF and MK Models for Degrees of Moneyness

S/X = 80/100	80%		
Term	5.00		
Coupon Rate (%)	3.28		
Model	TF	MK	
Credit Spread/Default Intensity (%)	4.00	8.36	
Equity Jump (%)		-70	-100
Constant	109.66	111.33	116.26
Inverse Function of Stock Price	110.21	112.14	114.92
Credit Spread/Default Intensity (%)	8.00	14.60	
Equity Jump (%)		-70	-100
Constant	97.68	102.94	114.59
Inverse Function of Stock Price	98.02	103.79	111.62
Credit Spread/Default Intensity (%)	12.00	57.28	
Equity Jump (%)		-70	-100
Constant	87.42	97.44	116.57
Inverse Function of Stock Price	88.49	98.94	112.05

¹⁰I selected the functional forms in Figures 1 and 2 numerically such that the price effect of the credit adjustment went to zero as the convertible debt was very deep in the money and to 100% when it was deep out of the money.

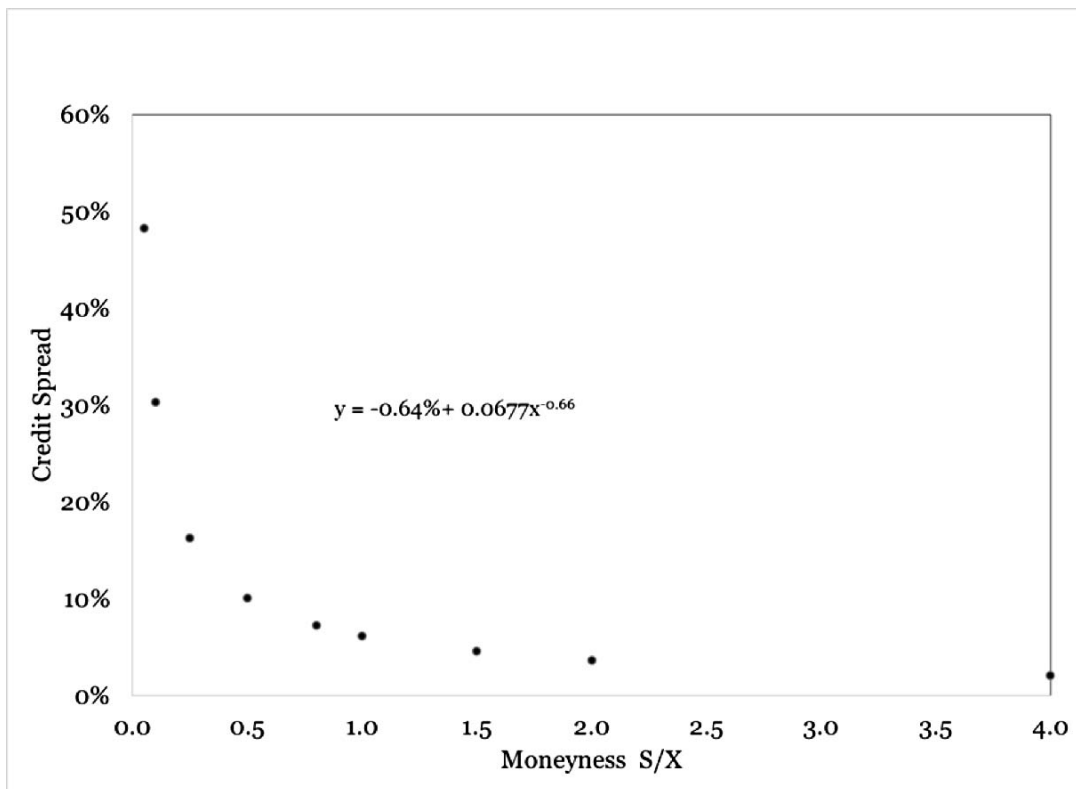


Figure 1
Credit Spread as an Inverse Function of Moneyness

2012 to early 2015. After eliminating all callable convertibles, to simplify the valuation, thirty-five issues remained. I calibrated all three models to market prices at the time of issuance of the debt.¹¹ The calibration process involved estimating the volatility of the underlying common stock and deriving the credit spreads (TF and GS models) or default intensity (MK model) that produced the price of the convertible debt at issuance. I used the calibrated models to forecast the price of the convertible debt one year after issuance, based on changes in the stock price, interest rate, time to maturity, and, possibly, the volatility of the common stock. I compared those forecasted values with the reported market prices of the convertible debt at that time.

I designate F_i and A_i as the forecast and actual percentage changes in value of each convertible debt over the one-year period following its issuance. I measure the forecasting performance in two ways. First, I calculate the average of the forecast errors, $(A_i - F_i)$, and test it for

¹¹I used the original models, where the credit risk did not vary with the stock price. To allow for the market to establish reliable values, I calibrated the models one week after issuance and, in a few cases where the market prices of the convertible debt were not yet available, either two or three weeks after issuance.

bias, and then I measure its standard deviation as an indication of accuracy. Second, I assess accuracy by estimating the following linear regression:

$$A_i = \hat{a} + \hat{b}F_i + e_i.$$

If these forecasts were perfect, I would find $\hat{a} = 0.0$, $\hat{b} = 1.0$, and the R^2 would be 100%.

I summarize the forecasting results in Table 7a for three different approaches to measuring volatility. The first approach was to measure the historic volatility of each security at the issuance date, calibrate the models, and then remeasure volatility at the forecast date, holding the credit spread or default intensity constant between issuance and the forecast. This approach produced negatively biased forecasts. The mean values of $(A - F)$ were -4.4%, -3.9%, and -3.6% for the TF, GS, and MK models, respectively. All three values were statistically significantly different from 0.0. I also found that the errors were negatively correlated with the initial measurement of volatility.¹²

This led to the second approach, which was to use both the initial volatility and the initial credit spread/default intensity at the forecast date, i.e., to not update

¹²I have not been able to identify an explanation for this result.

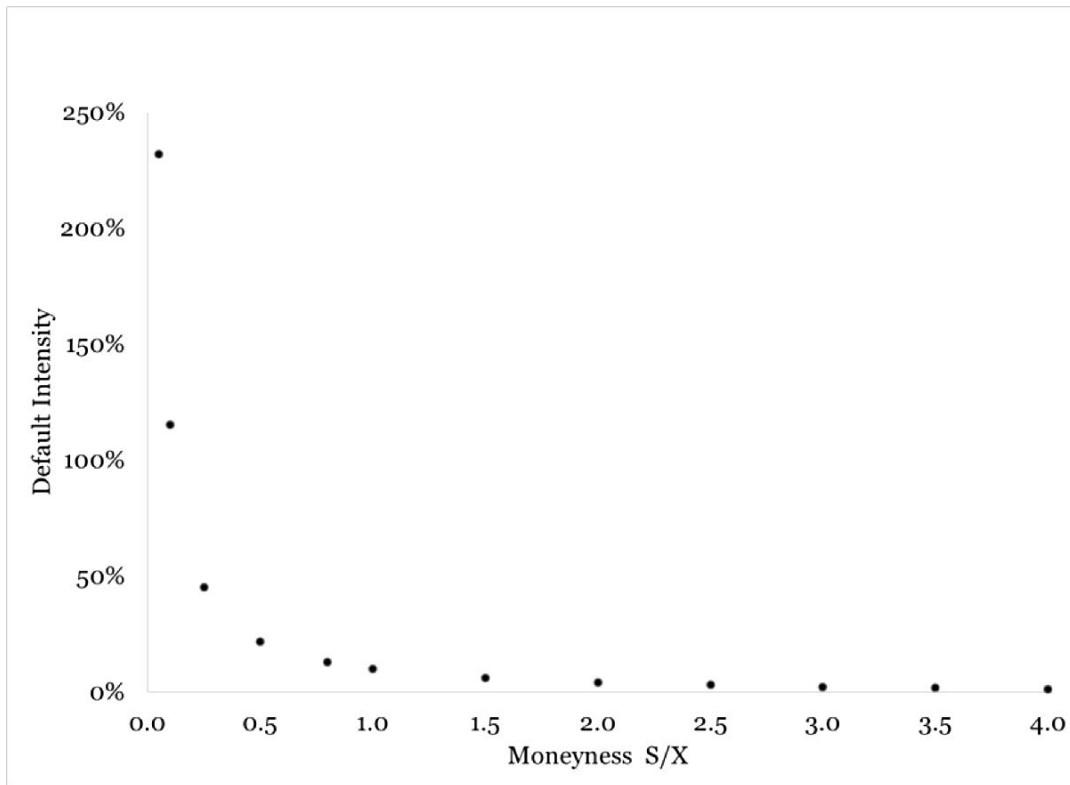


Figure 2
Default Intensity as a Function of Moneyness

the volatility estimate. The mean values of $(A - F)$ for this approach were 0.6%, 0.3%, and 0.0% for the TF, GS, and MK models, respectively, and none was statistically significantly different from 0.0. The standard deviations of $(A - F)$ for the second approach were also smaller than those for the first approach, indicating that it was not only unbiased, but also more accurate.

The results for the case with constant volatility suggest that consistency in the two inputs may be more important than the precision of their individual estimations. To

investigate that, I tested a third approach to volatility, namely, assuming a single constant volatility of 30% for all securities at both dates. This approach also produced unbiased forecasts with mean values of $(A - F)$ of 0.3%, 0.3%, and -0.2% for the TF, GS, and MK models, respectively. The accuracy of this third approach, as indicated by the standard deviations of the forecast error, is essentially identical to the accuracy of the second approach. This supports the idea that consistency in the two inputs is more important than the precision of their estimations.

Table 7a
Results of Price Forecasting Models

Alternative Volatility Choices	TF				GS				MK			
	$(A - F)$ (%)		Forecast Regression		$(A - F)$ (%)		Forecast Regression		$(A - F)$ (%)		Forecast Regression	
	Mean	Standard Deviation	\hat{b}	R^2 (%)	Mean	Standard Deviation	\hat{b}	R^2	Mean	Standard Deviation	\hat{b}	R^2 (%)
Two Different Volatilities	-4.39	8.02	0.91	90	-3.86	7.33	0.86	89	-3.55	6.76	0.93	87
One Volatility	0.65	6.93	0.97	93	0.29	5.90	0.92	93	-0.01	5.83	1.01	91
Volatility of 30%	0.30	6.41	0.88	93	0.27	6.48	0.88	91	-0.18	5.87	0.91	91

Table 7b
Calibration and Forecasting Results for Individual Issues Using Only One Volatility

Issuer	Issue Date	Tenor	At Issuance				Pricing One Year Later					
			Coupon (%)	S/X (%)	Volatility (%)	TF Credit Spread (%)	GS Credit Spread (%)	MK Default Intensity (%)	Convertible Market Price (\$)	TF (A - F) (%)	GS (A - F) (%)	MK (A - F) (%)
Anthem, Inc.	10/2/2012	30.0	2.75	82	30	1.5	1.4	2.8	130.19	-5.2	-5.9	-6.0
Brocade Comm. Systems	1/9/2015	5.0	1.38	74	30	2.4	2.2	4.0	93.50	1.9	2.2	2.1
Cardtronics plc	11/21/2014	6.0	1.00	75	34	4.3	3.8	8.4	97.63	-0.2	0.3	-1.2
Cobalt International Energy, Inc.	5/8/2014	10.0	3.13	75	40	7.4	5.9	14.3	78.63	1.9	5.0	4.2
Cobalt International Energy, Inc.	12/11/2012	7.0	2.63	72	74	13.2	9.3	26.0	86.81	-4.7	0.8	-3.8
Euronet Worldwide, Inc.	10/30/2014	29.9	1.50	75	34	2.7	1.7	5.6	129.19	7.7	5.8	5.1
Forest City Realty Trust, Inc	7/13/2012	6.1	4.25	65	44	10.7	9.4	21.6	110.00	3.5	2.6	2.1
Hornbeck Offshore Services, Inc.	8/8/2012	7.1	1.50	78	50	7.7	6.1	14.5	127.06	5.4	3.9	4.0
iStar, Inc.	11/13/2013	3.0	1.50	72	24	2.5	2.5	4.5	103.19	0.8	0.8	0.8
iStar, Inc.	11/7/2012	4.0	3.00	62	40	8.0	7.4	15.2	127.88	-3.1	-3.9	-4.3
Lam Research Corporation	5/10/2012	4.0	0.50	61	41	4.2	3.9	7.6	110.63	-5.4	-5.5	-5.7
Lam Research Corporation	5/10/2012	6.0	1.25	61	41	4.3	3.9	7.3	117.19	-9.1	-9.5	-8.4
LinkedIn Corporation	11/6/2014	5.0	0.50	78	41	4.4	3.8	8.7	108.50	1.9	2.1	0.8
MGIC Investment Corp.	3/7/2013	7.1	2.00	71	131	18.1	10.4	42.9	142.38	22.6	13.2	15.3
Micron Technology, Inc.	11/6/2013	30.0	3.00	64	40	6.9	4.0	16.4	126.50	14.3	5.9	7.6
Radian Group, Inc.	2/26/2013	6.0	2.25	99	70	14.0	9.1	30.1	159.75	5.1	2.1	3.8
Red Hat, Inc.	10/2/2014	5.0	0.25	79	31	2.4	2.2	4.5	124.44	-6.3	-6.4	-6.9
RPM International, Inc.	12/3/2013	7.0	2.25	76	24	0.0	0.0	0.0	113.38	3.7	3.9	3.7
SanDisk Corp.	10/24/2013	7.0	0.50	75	29	0.8	0.8	1.4	117.75	-3.4	-3.5	-3.3
SEACOR Holdings, Inc.	11/18/2014	14.0	3.00	60	20	3.6	3.5	7.6	79.25	2.9	3.6	2.8
Stone Energy Corp.	2/29/2012	5.0	1.75	75	64	14.4	11.0	29.4	91.06	-11.3	-7.1	-10.1
Stone Energy Corp.	5/8/2013	3.8	1.75	52	41	5.0	4.8	9.0	121.44	-0.4	-1.2	-0.2
Tesla Motors, Inc.	2/27/2014	7.0	1.25	70	58	8.2	6.3	15.4	83.69	6.1	8.1	6.9
Tesla Motors, Inc.	2/27/2014	5.0	0.25	70	58	6.8	5.5	13.2	86.63	8.1	9.5	6.6
Tesla Motors, Inc.	5/16/2013	5.0	1.50	74	53	10.8	8.8	20.4	179.94	-12.6	-16.7	-13.1
The Priceline Group, Inc.	6/3/2014	6.0	0.35	93	24	0.0	0.0	0.0	113.19	3.1	3.1	3.2
The Priceline Group, Inc.	3/6/2012	6.0	1.00	69	37	3.5	3.2	6.3	110.63	-3.7	-3.7	-3.5
The Priceline Group, Inc.	5/29/2013	7.0	0.35	60	33	2.0	1.8	3.6	120.81	-0.7	-1.3	-0.7
The Priceline Group, Inc.	3/8/2013	5.0	1.00	76	34	2.1	1.9	3.6	148.31	9.1	8.8	8.6
The Priceline Group, Inc.	8/14/2014	7.1	0.90	59	24	0.4	0.4	0.6	98.31	1.1	1.1	1.1
TTM Technologies, Inc.	12/16/2013	7.0	1.75	86	40	7.6	6.0	16.4	96.44	-1.4	0.1	-0.9
Twitter, Inc.	9/11/2014	5.0	0.25	66	54	5.6	4.7	10.4	88.31	-4.8	-3.0	-5.3
United States Steel Corp.	3/20/2013	6.0	2.75	78	45	9.4	7.9	18.1	127.94	-4.5	-5.5	-5.7
Verint Systems, Inc.	6/12/2014	7.0	1.50	78	25	2.5	2.3	4.6	116.06	1.1	0.8	1.5
Yahoo!, Inc.	12/5/2014	4.0	0.00	94	33	2.9	2.5	5.3	98.25	-1.0	-0.5	-1.4
Average		8.15	1.56	73	43	5.71	4.52	11.42	113.28	0.65	0.29	-0.01
Standard Deviation										6.93	5.90	5.83

When I measure forecast performance in terms of a regression of actual percentage change on forecast percentage change, I find the results are very similar for the three methods of measuring volatility and for the three models. Using two different volatilities produces slightly lower R^2 values for the TF and GS models, and given that its forecasts are biased, I would tend to reject the idea of trying to update the volatility when holding the measure of credit risk constant. The TF and GS models have slightly higher R^2 values than the MK model, but the differences are so small that all three models perform essentially identically. This conclusion is further reinforced by the fact that the pair-wise R^2 values between the three forecasts using a single volatility are greater than 99.5%.

In Table 7b, I provide summary statistics and individual results for the thirty-five convertible debt securities in this sample for the analysis using a single volatility. The average tenor of the convertibles at issuance was 8.15 years, the average coupon was 1.56%, the average degree of moneyness was 73%, and the average volatility was 43%.¹³ The average credit spreads were 5.85% for the TF model and 4.61% for the GS model. It appears that the credit spread for the TF model is consistently larger than that for the GS model. This means that in the context of partitioning the debt value for financial reporting purposes, the TF model will consistently identify a smaller debt value and higher yield than the GS model. The average default intensity was 11.42%. In general, this value translates to an effective yield that is higher and bond value that is lower than either of the credit spread models.

Conclusions

In three important respects, the convertible debt models are very similar. First, the base case example illustrated that the three models produce quite similar prices for the same inputs and a jump in the stock price of -70% in the event of default. Second, I showed that there is little effect on the value of the convertible if the credit spread or default intensity is a function of stock price. Third, I provide evidence that the three models perform well in forecasting convertible debt prices one year after calibrating each of the models to market data. In the context of estimating convertible debt prices for non-traded securities for financial reporting purposes, this is reassuring. I also identified one important difference in the comparative responses of the TF and GS models and

the MK model to changes in credit quality. Contrary to expectations, the convertible debt price produced by the MK model increased when credit quality decreased. This is because there is a complex relationship between credit quality and volatility of equity, such that the decrease in credit quality increased stock price volatility and increased the value of the conversion feature. This observation highlights an important underlying difference in the assumptions of the TF and GS models as compared to the MK model. The TF and GS models assume that the stock price follows a geometric Brownian motion, while the MK model assumes the stock price follows a jump-diffusion process. This difference should be considered in estimating the volatilities to use in each model. In particular, volatility estimation is more complex if you assume stock prices follow a jump-diffusion process. That said, on the whole, I find that the models are sufficiently similar that all three can reasonably be used to estimate the value of nontraded convertible debt and will provide very similar valuations of the convertible debt. If these models are used to partition the debt component, the debt component will generally be largest for the TF model and smallest for the MK model.

References

- Cox, J. C., S. A. Ross, and M. E. Rubinstein. "Option Pricing: A Simplified Approach." *Journal of Financial Economics* 3 (September 1979):229-263.
- Bardhan, I., A. Bergier, E. Derman, C. Dosembet, and I. Kani. 1994. "Valuing Convertible Bonds as Derivatives." In *Quantitative Strategies Research Notes*. New York: Goldman Sachs.
- de Spiegeleer, J., and W. Schoutens. 2011. *The Handbook of Convertible Bonds: Pricing, Strategies and Risk Management*. West Sussex, United Kingdom: John Wiley & Sons Limited.
- Hull, J. 2011. *Options, Futures and Other Derivatives*. 8th ed. Upper Saddle River, NJ: Prentice-Hall.
- Jarrow, R. A., and A. Rudd. 1983. *Option Pricing*. Homewood, IL: Irwin.
- Milanov, Krasimir, and Ognyan Kounchev. June 2012. *Binomial Tree Model for Convertible Bond Pricing within Equity to Credit Risk Framework*. Working Paper of the Institute of Mathematics and Informatics. Sofia, Bulgaria: Bulgarian Academy of Science.
- Tsiveriotis, K., and C. Fernandes. "Valuing Convertible Bonds with Credit Risk." *Journal of Fixed Income* 8 (September 1998):95-102.
- Zabolotnyuk, Y., R. Jones, and C. Veld. "An Empirical Comparison of Convertible Bond Valuation Models." *Financial Management* 39 (Summer 2010):675-706.

¹³For two of the securities, the historical volatility implied negative credit spreads and default intensity, and in those two cases, I adjusted the volatilities until the credit spreads and default intensity were 0.0%.

From the Chair

William A. Johnston, ASA

Hello again! Time certainly goes by fast, and this is my last letter as Chair of the Business Valuation Committee. Thank you for the opportunity to serve.

It's Not Always Where You End, It's the Journey . . .

Often I am asked the benefits of getting involved with ASA. There are countless tangible benefits that are obvious: the networking, the education, building your credentials, etc. However, let me tell you another reason—the challenges you face and how you deal with them. You learn a lot from these experiences and are a much better person and professional as a result.

So, you can choose to sit in your office and not expand your horizons, or you can try new endeavors. When you take on these challenges, sometimes you are going to fail. Sometimes you do not know how you will get there or what is waiting for you when you do. It is really the journey getting there that I find most rewarding and what has made me (I hope) a better person and professional. Get outside of your comfort zone and push yourself, and good things will happen as a result—trust me!

Fair Value Quality Initiative—CEIV™ Credential

The CEIV™ has launched. The three Valuation Profession Organizations involved with this effort begun offering the credential Certified in Entity and Intangible Valuations™ or CEIV™. Do not worry if you do not have the credential yet, as many people are just starting the process or evaluating how to proceed.

Also remember that anyone can get the credential, regardless whether you have a BV credential or not. The

pathway is longer, of course, but it can nonetheless be done. Please be in touch with any questions.

Enhanced Equivalency for CFA Designation

I want to keep mentioning that the BV committee approved increased equivalency for individuals with the CFA designation, provided that they have at least five years of full-time business valuation experience. Now, if an individual has at least five years of full-time business valuation experience and the CFA designation, he or she is not required to take ASA's BV 201 to BV 204 classes. This decision was arrived at after a thorough review and comparison of the CFA designation versus the ASA designation. Candidates will still need to meet the other requirements (e.g., submitting a report and our USPAP/ethics requirements). If you are interested in learning more about this, please be in touch with me directly (billj@empireval.com) or contact ASA.

Last Words

I could not be more pleased to have such a high-quality professional follow me as Chair as Jeff Tarbell. Our committee could not be in any better hands, and if he needs my help I will be there to provide it.

What a ride! What sets ASA apart the most in my opinion is how much our members really care about the profession and the quality of the work that they do—you take it so seriously. Continue being proud and being proud of ASA!

The Business Valuation Committee

Chair	William A. Johnston, ASA – Empire Valuation Consultants, LLC
Vice Chair	Jeffrey S. Tarbell, ASA – Houlihan Lokey
Secretary	Erin Hollis, ASA – Marshall & Stevens, Inc.
Treasurer	Kenneth J. Pia, Jr., ASA – Meyers, Harrison & Pia, LLC
Past Chair	Robert B. Morrison, ASA – Morrison Valuation & Forensic Services, LLC
Members At-Large	Gregory Ansel, ASA – Financial Strategies Consulting Group LLC Arlene Ashcraft, ASA – Columbia Financial Advisors, Inc. KC Conrad, ASA – American Business Appraisers Matthew R. Crow, ASA – Mercer Capital Nancy M. Czaplinski, ASA – Duff & Phelps Erin Hollis, ASA – Marshall & Stevens, Inc. Bruce Johnson, ASA – Munroe, Park & Johnson, Inc. Curtis T. Johnson, ASA – Ernst & Young Timothy Meinhart, ASA – Willamette Management Associates Susan Saidens, ASA – SMS Valuation & Financial Forensics Ronald L. Seigneur, ASA – Seigneur Gustafson LLP Gary R. Trugman, ASA – Trugman Valuation Associates, Inc. Mark Zyla, ASA – Acuitas, Inc.
Ex Officio	James O. Brown, ASA – Perisho Tombor Brown, PC R. Chris Rosenthal, ASA – Ellin & Tucker Linda B. Trugman, ASA – Trugman Valuation Associates, Inc.
Emeritus	Jay E. Fishman, FASA – Financial Research Associates Shannon P. Pratt, FASA – Shannon Pratt Valuations, LLC
HQ Advisor	Bonny Price

BUSINESS VALUATION REVIEW™ —Published quarterly by the Business Valuation Committee of the American Society of Appraisers.
Editor: Daniel L. McConaughy, PhD, ASA. Managing Editor: Rita Janssen, rjanssen@allenpress.com.

INTERNATIONAL STANDARD SERIAL NUMBER—The Library of Congress, National Serials Data Pro = ISSN 0897-1781.

SUBSCRIPTION RATES—PRINT AND ELECTRONIC ACCESS: Members of American Society of Appraisers—\$60 if paid with annual dues; all others—\$140 per year. Direct any questions and/or subscription changes to Bonny F. Price (703) 733-2110 or bprice@appraisers.org

MANUSCRIPTS—Authors should submit their articles (including charts, graphs, pictures, etc.) for publication in *Business Valuation Review™* at <http://www.editorialmanager.com/bvr/>. Follow the new user registration login instructions, at the Register Now hotlink. Since we use a double-blind peer review system, authors should provide the author's name (as it would appear if published), and a short one-paragraph description of professional activities when completing the registration process. An abstract should appear at the beginning of the paper submitted for review. Authors are asked to not submit manuscripts currently under review by other publications or that have been published elsewhere. All manuscripts are reviewed by the Editorial Review Board and may be accepted, rejected, or returned for additional work. Editor's notes may be included with the printing of accepted articles. All authors will be so notified. No manuscripts will be returned, and no critiques or reasons for rejection will be given for material not selected for publication.

CLASSIFIED ADS—Accepted for publication in one issue only and should reach the Publisher no later than the 10th of the month prior to publication. Rates: full page—\$500, 2/3 page—\$375, 1/2 page—\$300, 1/3 page—\$200, 1/4 page—\$150. Payable in U.S. funds. Direct any questions to Bonny F. Price (703) 733-2110 or bprice@appraisers.org

LETTERS TO THE EDITOR & COMMENTS—Letters from readers are encouraged on any valuation subject. *BVR™* reserves the right to edit letters for length, taste and/or clarity. No guarantee of publication is given. Please direct critique or comments to the Editor. All Letters to the Editor should be submitted at <http://www.editorialmanager.com/bvr/>.

STATEMENTS & OPINIONS—Statements expressed herein by the authors and contributors to *Business Valuation Review™* are their own and not necessarily those of the American Society of Appraisers or its Business Valuation Committee.

Copyright, © American Society of Appraisers—2017

All rights reserved. For permission to reproduce in whole or part, and for quotation privilege, contact International Headquarters of ASA. Neither the Society nor its Editor accepts responsibility for statements or opinions advanced in articles appearing herein, and their appearance does not necessarily constitute an endorsement.



Published by the American Society of Appraisers
Copyright © 2017 American Society of Appraisers